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Comparison of forecasting precision with ARCH &G ARCH models by using the prices of Iraqi crude oil export values

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Abstract

Many time series data are characterized by a large number of random changes, which makes them suffer from heteroscedasticity, where classical analyzes require homogeneity of variance, us research is interested in to study of time series that suffer from Volatility in the form of periods of high and low Which is concerned with the element of risk or uncertainty in the market because it's importance in the subject of building dynamics models of time series , and then the use of those models in forecasting the values of the phenomenon studied, In this research were used the conditional autoregressive models of heteroscedasticity when the errors follow a normal distribution that takes into account the Volatility that occur in prices, it began with the diagnosis stage using the ARCH effect test to detect The problem of heteroscedasticity in the remainder of the estimated model, followed by the estimation stage, then examining the suitability of the model by using several measures for precision, and then the forecasting process. for using the values of the prices of Iraqi crude oil exports for a series of (136) observations for the period between the period 1/1/2011 to 4/1/2022. It appears that the best model GARCH(1,1) because it achieved the lowest values of the precision measures .

Keywords: heteroscedasticity, ARCH & GARCH models , stationary Tests , precision measures

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مقارنة دقة التنبؤ باستخدام نماذج ARCH وG ARCH باستخدام أسعار صادرات النفط الخام العراقي

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المستخلص

تتميز العديد من بيانات السلاسل الزمنية بوجود عدد كبير من التغيرات العشوائية، مما يجعلها تعاني من تباين غير متجانس. في حين تتطلب التحليلات الكلاسيكية تجانس التباين، يهتم بحثنا بدراسة السلاسل الزمنية التي تعاني من التقلبات على شكل فترات من الارتفاع والانخفاض، وهو ما يرتبط بعنصر المخاطرة أو عدم اليقين في السوق نظرًا لأهميته في بناء نماذج ديناميكية للسلاسل الزمنية، ثم استخدام هذه النماذج في التنبؤ بقيم الظاهرة المدروسة. في هذا البحث، تم استخدام نماذج الانحدار الذاتي الشرطي للتباين غير المتجانس عندما تتبع الأخطاء توزيعًا طبيعيًا يأخذ في الاعتبار التقلبات التي تحدث في الأسعار. بدأ البحث بمرحلة التشخيص باستخدام اختبار تأثير ARCH للكشف عن مشكلة التباين غير المتجانس في بقية النموذج المقدر، تلتها مرحلة التقدير، ثم فحص مدى ملاءمة النموذج باستخدام عدة مقاييس للدقة، ثم عملية التنبؤ. باستخدام قيم أسعار صادرات النفط الخام العراقي لسلسلة من (136) مشاهدة للفترة بين 2011/1/1 و 2022/1/4، يبدو أن أفضل نموذج هو GARCH (1,1) لأنه حقق أقل قيم لمقاييس الدقة.

الكلمات المفتاحية: عدم تجانس التباين، نماذج ARCH و GARCH، اختبارات الاستقرار، مقاييس الدقة

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معلومات البحث
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Introduction

The time series forecasting process is considered one of the most important methods of statistical analysis, which is used in many different fields of applied sciences. The analysis of financial time series appeared a clear development at the beginning of the second half of the twentieth century, after the two scientists (Box-Jenkins) presented a modern methodology, which proved highly efficient in modern analysis of time series, as it contained statistical theories, methods, and graphic and computational means used in identifying the model. estimating its parameters, diagnosing and using it to forecasting future observations represented by ARMA models, Now, what is known for ARMA models is that they cannot explain the Volatility and changes in some phenomena that are characterized by their large time Volatility. In addition to this, these linear models have some assumptions about random errors, which is the linear structure and the homogeneity of the variance of errors may not be achieved on the practical side, and therefore they cannot explain these Volatility. To overcome this problem, ARCH and GARCH models have been proposed, as these models are intended for variance modeling and are more commonly used in financial statements. It is also known that in the Classical standard analysis, the variance of the random limit is assumed to be constant over time, but in the financial statements as well as other economic data, this condition is often not fulfilled due to the appearance of fluctuations in the variance in the periods of the series. Therefore, this modern technique came to model the behavior of variance, where the scientist Engle in 1982 proposed a conditional autoregressive model of heteroscedasticity (ARCH), the aim of which is to analyze and control the current time Volatility as a function of the real sizes of random errors in the previous time interval. However, this model requires many parameters to be estimated to build the following model describing the process of time Volatility in a manner commensurate with the phenomenon under study. In order to overcome these difficulties, a more comprehensive model was proposed, which is the generalized autoregressive conditional model of heteroscedasticity (GARCH), which was proposed by Bollerslev in 1986, and this type of modeling led to a major transformation in applied econometrics.

2. The Research problem

In fact, the problem of research is that heteroscedasticity of the random errors of the estimated models of the time series. Therefore, the use of nonlinearity models (ARCH & GARCH) was used to in diagnosing the efficient model because the linear models became unable to do so.

3. The Research Aim

This research aims to review the theoretical aspects of the conditional autoregressive models of heteroscedasticity(ARCH, GARCH), as well as to formulate the best model for describing variables in prices among the above models in the event of Volatility in prices. As a result, the best suitable model for the data will be diagnosed from a set of proposed models and an estimate of its parameters will be provided. The parameters of the model and examine the suitability of the model, and in the last stage, it includes making forecasting for the future prices.

4. The theoretical side

In this side, autoregressive models are studied conditional on the heteroscedasticity, as it includes the function of these models, in addition to the stage of diagnosis and the estimation, with the study of some criteria to determine the rank, in addition to the tests for this model.

4-1 The Autoregressive Conditionally heteroscedasticity Model (ARCH)

This model was presented by the scientist Robert Engle in a paper on estimating the variance of inflation in the United Kingdom published in 1982. It is a conditional autoregressive model with heteroscedasticity for random errors and is characterized by a constant conditional mean and conditional variance known by the equation of non stationary Volatility, and it is represented by: The ARCH model predicts that error boxes that depend on the past with periods of slowing back, and that these Volatility in the time series can be described according to the following equations, which represent the ARCH mode [1,2]:

$$\mathbf{z_t = \mu + e_t} \qquad \textbf{Mean equation} \quad (1)$$

$$e_t = \sigma_t \varepsilon_t \qquad \varepsilon_t \sim \mathrm{iidN}(0,1)$$

$$\boldsymbol{\sigma_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \cdots + \alpha_\rho e_t^2} \qquad \mathbf{(2)}$$

When p = 1 write the model according to the following equation:

$$\boldsymbol{\sigma_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2} \qquad \mathbf{(3)}$$

So that (α_0 > 0), (α_i ≥ 0, I > 0) represents the coefficients of the model and equation (1) represents the mean equation.

And the expansion of (p) values in the ARCH model results in negative values for (α), which leads to a contradiction with the assumptions of the ARCH.

4-2 Generalized Autoregressive Conditionally

$z_t = \mu + e_t$ Mean quation

$e_t = \sigma_t \varepsilon_t$ $\varepsilon_t \sim iidN(0,1)$

generalized model called the generalized autoregressive model conditional of heteroscedasticity (GARCH), which was proposed by the scientist Bollerslev in 1986 and often requires many parameters to describe the process of heteroscedasticity in the series accurately. This model consists of the fixed term and the values of the residual squares and the variance values for the previous periods and is written according to the following equation [3,4]:

$$\boldsymbol{\sigma_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \cdots + \alpha_\rho e_{t-\rho}^2 + \beta_1 \sigma_{t-1}^2 + \cdots + \beta_q \sigma_{t-q}^2} \qquad \mathbf{(4)}$$

So q ≥ 1, p ≥ 1, where p represents the slowdown period of the squares of errors, q represents the slowing period of the conditional variance, (z_t) represents the return series, which is a stationary series and μ represents the average of the series of returns, (ε_t) represents the sequence of errors Random .

And the model stationary condition, β_j , $\alpha_i \geq 0$, $\alpha_0 > 0$, $\sum_{i=1}^{\rho} \alpha_i + \sum_{j=1}^{q} \beta_j < 1$, represents the coefficients. model

5- Stages of building ARCH & GARCH models

5-1 The identification

One strongly stationary time series is one of the important conditions for its modeling, and many time series are non-stationary on average and have the property of volatility. To detect the stationary of the time series, the graph of the original series is used. And use the differences to make them stationary in the mean , Transforming the non stationary series into the return series with the following structure :

$$Z_t = \ln(x_t) - \ln(x_{t\_1}) \qquad (5)$$

5-1-1 stationary Test (Unite Root)

When analyzing the time series, its stationary must be tested. For this purpose, the extended unit root test (Dickey-Fuller) is used, which is written in abbreviation (ADF), which is one of the most powerful tests to detect the stationary time series for each variable separately. The test is done according to the following formula [5,6]:

$$\Delta Z_t = \alpha + \beta_t + \gamma Z_{t-1} + \sum_{j=1}^{m} b_j \Delta Z_{t-j} + \varepsilon_t \qquad (6)$$

That $z_t$represents the series of returns,ε_t represents the random error and is not self-correlated, Δ represents the first difference, $(\alpha,\beta, \gamma ,,b_j)$ represents the model parameters, $\varepsilon_t$represents the error element and that the (ADF) test is done under the following hypothesis:

$H_0: \Upsilon = 0$ Yield of series returns possesses a unit root (non stationary yield of series returns)

$H_1 : \Upsilon < 0$ none Yield series of returns possesses a unit root (stationary yield of series returns)

The test statistics are given according to the following equation:

$$\text{ADF} = \frac{\hat{\Upsilon}}{se(\hat{\Upsilon})} \qquad (7)$$

$se(\hat{\gamma})$represents the standard error, and in general, if the p-value is less than (0.05), this indicates that the alternative hypothesis is accepted, that is, there is no unit root and that the variable is stationary.

5-1-2 ARCH – Test

The ARCH Test is used to show whether the problem heteroscedasticity of random error is real or not. This test is based on the Langrage multiplier LM, which was proposed by the scientist Engle in 1982, where this test is applied to estimate errors according to the equation $(e_t = z_t - \mu_t)$. Where the squares of errors are used to do regression on a constant and the squares of errors from previous periods, as in the following equation [7,8]:

$$e_t^2 = \alpha_0 + \alpha_1 e_{t-1}^2 + \cdots + \alpha_\rho e_{t-\rho}^2 \qquad (8)$$

After finding the coefficient of determination resulting from estimating the equation (8), the test statistic is calculated as follows:

$$LM = NR^2 \qquad \sim x_\rho^2 \qquad (9)$$

Where R^2 represents the coefficient of determination, N represents the sample size, ρ represents the degree of the model, since the test statistic follows a Chi-Square distribution with a degree of freedom of p, and the test is carried out according to the following hypothesis:

$H_0: \alpha_1 = \alpha_2 = \cdots = \alpha_\rho = 0$

$H_1 : \alpha_1 \neq \alpha_2 \neq \cdots \neq \alpha_\rho \neq 0$

And under the null hypothesis of homogeneity of variance of random errors, if (p-value < 0.05) we reject the null hypothesis, i.e. there is a problem of heteroscedasticity of random errors and, therefore, acceptance of the ARCH model. Meaning there is an effect ARCH model.

5-2. Estimation the parameters of the model

This step comes after the step of figuring out the right model for the time series data. There are different ways to estimate this step [9,10].

$$\mathbf{L}(\mathbf{a}, \emptyset, \boldsymbol{\mu}, \boldsymbol{\theta}, \boldsymbol{\sigma}_a^2) = (2\pi\boldsymbol{\sigma}_a^2)^{\frac{-N}{2}} \exp\left(\frac{-1}{2\sigma_a^2}\sum_{t=1}^{n} \alpha_t^2\right) \qquad (10)$$

The logarithm of the conditional maximum possibility function is as follows:

$$\mathbf{LnL}(\emptyset, \boldsymbol{\mu}, \boldsymbol{\theta}, \boldsymbol{\sigma}_a^2) = -\frac{N}{2}\ln(2\pi\boldsymbol{\sigma}_a^2) - \frac{s(\emptyset,\mu,\theta)}{2\sigma_a^2} \qquad (11)$$

Where the conditional sum of squares function is:

$$\mathbf{Ln}(\emptyset, \boldsymbol{\mu}, \boldsymbol{\theta}) = \sum_{t\to\infty}^{N} [E(a/\emptyset, \mu, \theta]^2 \qquad (12)$$

5.3. Model selection criteria: This is done by using specific standards such as (AIC, H-Q, SIC)

5-4-Forecasting

There are many evaluation criteria to measure the prediction accuracy within the sample, because some research work has found that the chosen model is not necessarily the best model for forecasting. The best predictive model is the one that gives the least error (minimize the error) and the criteria used [11,12].

5-3-1 Root Mean Square Error

It takes the following form:

$$\mathbf{RMSE} = \sqrt{\frac{\sum_{t=1}^{T}(r_t^2 - \hat{\sigma}_t^2)^2}{T}} \qquad t = 1, 2, \ldots, T \ \ldots \qquad (13)$$

5-3-2. Mean Absolute Error

It is expressed in the following form:

$$\mathbf{MAE} = \frac{\sum_{t=1}^{T}|r_t^2 - \hat{\sigma}_t^2|}{T} \qquad t = 1, 2, \ldots, T \qquad (14)$$

5-3-3. Mean Absolute percentage Error(MAPE **)**

$$\text{MAPE}=\frac{1}{T}\sum_{t=1}^{T}\frac{|r_t^2-\hat{\sigma}_t^2|}{r_t} \quad t=1,2,\dots ,T \;\dots\dots \qquad (15)$$

r_t^2: Square time series observations, $\hat{\sigma}_t^2$: The estimated conditional variance obtained from the fit of the autoregressive conditional heteroscedasticity models

6. Application side:

It is also known that financial time series are subject to severe fluctuations as a result of being affected by political factors, wars, climate change, and others. Such fluctuations negatively affect the construction of forecast models for financial chains, which arises due to the problem of heterogeneity of variance for those models. A sample was taken to study the impact of these Volatility. From the prices of the monthly values of Iraqi crude oil exports by (136) views between the periods of 1/1/2011 and 1/4/2022, as the time series for oil prices was drawn as shown in the following figure (1):

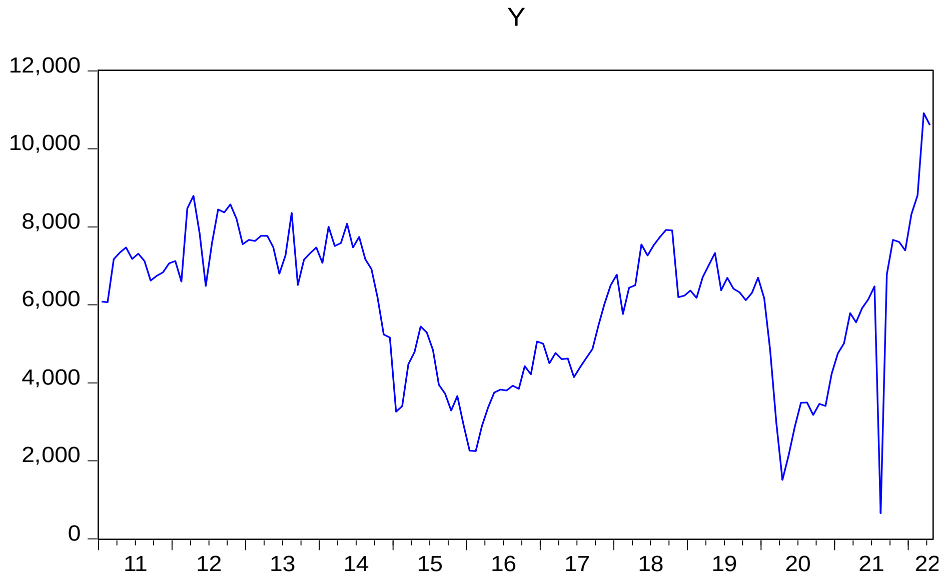


Figure (1) represents the time series of the sourced Iraqi oil price data

We note from the figure that the time series of oil prices is characterized by instability in the average and variance, and to make the series stable, we will convert it into a series of returns that represents the first difference in the logarithm of the series observations, as in Figure (2).

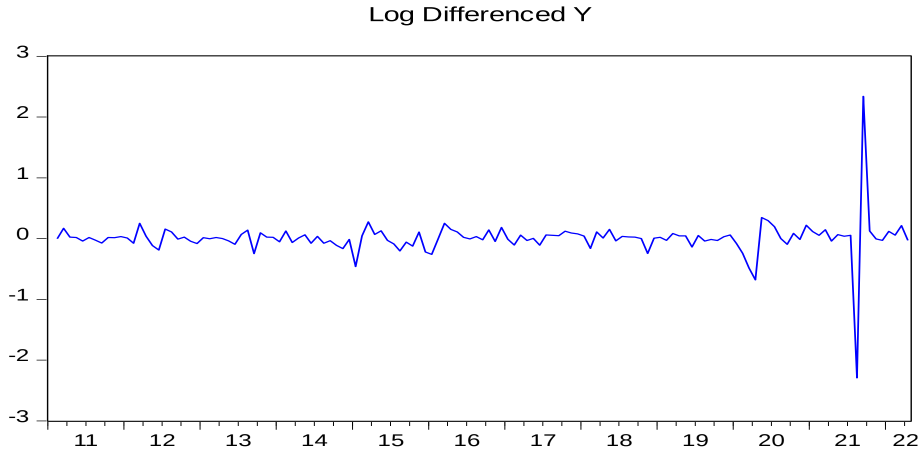


Figure (2) represents the series returns diagram of the Iraqi sourced oil price data

First: The Test of data

To test of data, the unit root test (Augmented Dickey Fuller) was used to determine whether the yield series returns was stationary or non stationary. The following results were obtained, as shown in the following table (1):

Table No. (1) Unit Root Test (Augmented Dickey Fuller) for Yield series returns

			t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic			-14.32893	0.0000
Test critical values:	1% level		-3.479656	
	5% level		-2.883073	
	10% level		-2.578331	

The table shows the stationary of time series, where the results showed that the value of (p-value=0.000) is less than (0.05), which is evidence of the stationary of the time series.

Second: Building an ARCH/GARCH models

First, we are working on building a time-series model, and after several attempts, and depending on the criteria of differentiation in choosing the best model, the autoregressive model of order (1), AR(1) was adopted as the best model that can be reconciled to predict the time series, because it achieved the lowest value on the differentiation criteria (AIC, H-Q, SIC). The results of estimating the model AR (1) were clarified in Table No. (2), which shows the significance of the model parameter $\emptyset_{1}$ because the probability value (p-value = 0.000) is less than the level of significance α=0.01, which indicates that the estimated relationship is significant. The results also showed that the value of the coefficient of determination is, as shown in the following table (2):

Table (2) Model Estimation Results AR(1)

Dependent Variable: Y		
Method: Least Squares		
Date: 07/06/22 Time: 08:13		

Sample (adjusted): 2011M02 2022M04				
Included observations: 135 after adjustments				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	747.1171	282.3862	2.645728	0.0091
Y(-1)	0.880255	0.045324	19.42149	0.0000
R-squared	0.739315	Mean dependent var		5992.673
Adjusted R-squared	0.737355	S.D. dependent var		1868.620
S.E. of regression	957.6471	Akaike info criterion		16.58154
Sum squared residuals	1.22E+08	Schwarz criterion		16.62458
Log likelihood	-1117.254	Hannan-Quinn criter.		16.59903
F-statistic	377.1944	Durbin-Watson stat		2.270440
Prob(F-statistic)	0.000000			

Third: Residual analysis

Figure (3) The graph of the residual series shows that there is high and low volatility, and this is an indication of the conditional heterogeneity phenomenon of variance, that is, the presence of the influence of the (ARCH & GARCH) family in the mode.

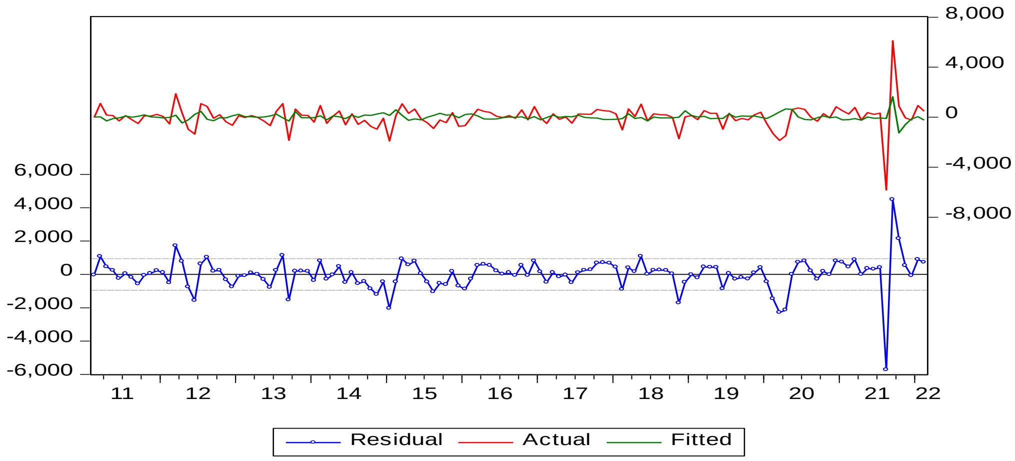


Figure (3) The graph of the residual series of the estimated model

Fourth: ARCH effect

The test is carried out according to the following null hypothesis:

H_0 : There is no ARCH effect

H_1: There is an ARCH effect

Table (3) shows the results of the ARCH effect test, in which the obs R-Square statistic value appears to have reached (28.86006) with a p-value = 0.000 This value is less than the level of significance and therefore the null hypothesis is rejected. This means that there is an ARCH effect

Table (3) Results of the heterogeneity test (ARCH TEST)

Heteroscedasticity Test: ARCH					
F-statistic	36.23293	Prob. F(1,132)			0.0000
Obs*R-squared	28.86006	Prob. Chi-Square(1)			0.0000
Test Equation:					
Dependent Variable: RESID^2					
Method: Least Squares					
Date: 07/04/22 Time: 03:21					
Sample (adjusted): 2011M03 2022M04					
Included observations: 134 after adjustments					
Variable	Coefficient	Std. Error	t-Statistic		Prob.
C	484949.8	275660.2	1.759231		0.0809
RESID^2(-1)	0.464070	0.077096	6.019380		0.0000
R-squared	0.215374	Mean dependent var			904518.1
Adjusted R-squared	0.209429	S.D. dependent var			3472231.
S.E. of regression	3087300.	Akaike info criterion			32.73831
Sum squared resid	1.26E+15	Schwarz criterion			32.78156
Log likelihood	-2191.466	Hannan-Quinn criter.			32.75588
F-statistic	36.23293	Durbin-Watson stat			1.797289
Prob(F-statistic)	0.000000				

Fifth: Diagnosing the degree of influence of the ARCH-GARCH family.

Multiple models have been tested for the purpose of diagnosing the degree of influence in the model and whether this effect can be represented by the ARCH model only or goes beyond that to models from other families as well as the rank of each of them, as it is known, the test will be on the model that achieves the lowest value for the three differentiation criteria (AIC, SIC and H-Q), The results are shown in Table (4).

Table (4) Diagnostic Results for (GARCH)

model	AIC	SIC	H_Q
ARCH(1)	16.37629	16.46237	16.41127
GARCH (1,1)	16.31976	16.42737	16.36349
GARCH(1,2)	16.35621	16.48533	16.40868
GARCH(2,2)	16.37703	16.52767	16.43825
GARCH(2,1)	16.36897	16.49810	16.42139

From observing the results in the above table, we note that the best model represents the effect of ARCH at order (1) and the effect of GARCH at order (1), so the model is (ARCH-GARCH (1,1) because it achieved the lowest values of the three criteria.

Sixth: Test the efficiency of the model

The ARCH-Test is conducted on the final estimated model to find out if the model still suffers from the effect of the presence of ARCH family models in the residual series according to the following hypothesis test:

H_0 : There is no ARCH effect

H_1 : There is an ARCH effect

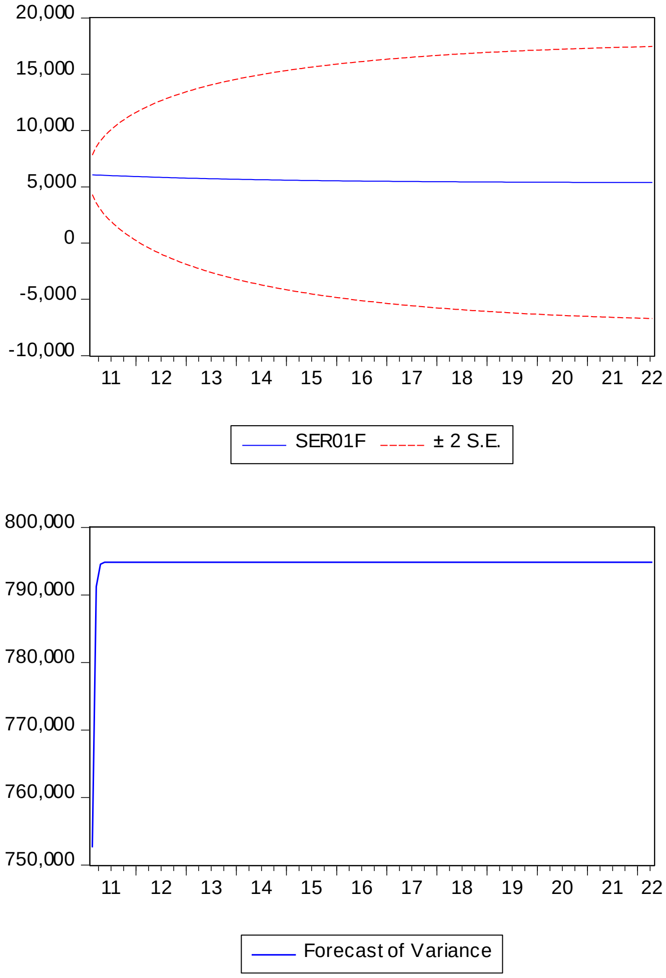
The results of the test were presented in Table No. (5), which shows that the probabilistic value of the test statistic was greater than the level of significance, and therefore the null hypothesis is accepted at the level of significance of 5%, and it is judged that there is no effect of the ARCH family in the estimated model.

Table (5): ARCH-Test

Heteroscedasticity Test: ARCH				
F-statistic	0.498031	Prob. F(1,132)		0.4816
Obs*R-squared	0.503677	Prob. Chi-Square(1)		0.4779
Test Equation:				
Dependent Variable: WGT_RESID^2				
Method: Least Squares				
Date: 07/06/22 Time: 05:53				
Sample (adjusted): 2011M03 2022M04				
Included observations: 134 after adjustments				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.929877	0.392052	2.371819	0.0191
WGT_RESID^2(-1)	0.061308	0.086874	0.705713	0.4816
R-squared	0.003759	Mean dependent var		0.990601
Adjusted R-squared	-0.003788	S.D. dependent var		4.419316
S.E. of regression	4.427680	Akaike info criterion		5.828441
Sum squared resid	2587.774	Schwarz criterion		5.871693
Log likelihood	-388.5056	Hannan-Quinn criter.		5.846017
F-statistic	0.498031	Durbin-Watson stat		1.995377
Prob(F-statistic)	0.481610			

Seven: Forecasting

The final step of each time-series model is to predict the values of the studied phenomenon. Figure (4) shows the estimated model's prediction of the residual variances. This shows that the estimators were within the confidence limits at a significance level of 0.95, which shows how good the estimators of the model are.



Forecast: SER01F	
Actual: SER01	
Forecast sample: 2011M01 2022M04	
Adjusted sample: 2011M02 2022M04	
Included observations: 135	
Root Mean Squared Error	1851.439
Mean Absolute Error	1582.861
Mean Abs. Percent Error	35.91355
Theil Inequality Coefficient	0.156317
Bias Proportion	0.053165
Variance Proportion	0.817965
Covariance Proportion	0.128870
Theil U2 Coefficient	0.565623
Symmetric MAPE	28.66607

Figure (4) shows the predicted value of oil prices

7. Conclusions

1. The time series of the monthly prices of exported crude oil for the period from (2011-2022) witnessed a large fluctuation in prices. We conclude from this the non stationary of time series and after taking the first difference, the series became stationary.
2. We concluded that the AR (1) model to represent the exported crude oil price series returns is the best model according to the three criteria (AIC, SIC and H-Q) for several models nominated by ARIMA.
3. We reached the non stationary of series in variance because the Heteroscedasticity.
4. We concluded that the model GARCH(1,1) is the best model.

8- Recommendations:

1. Use of other models in the estimation as TGARCH, EGARCH models.
2. Using other methods of estimation instead of OLS .

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