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Analysis and Investigation of fractional time Schrödinger equations

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Abstract

In this paper , the time-fractional nonlinear Schrödinger differential equations are solved using Jacobi's space-time spectral coherence technique (JSC method) under suitable beginning and boundary conditions. The examined issue is first converted into a related system of single weak kernel non-linear integral partial differential equations with the definition and characteristics of integral and fractional derivative operators. Thus the issue reduces to a collection of nonlinear algebraic equations by integrating the system associated with integral differential equations in both spatial and temporal variables together with the approximation of the integral in the equation using Jacobi Gauss quadratic method. Several reliable iterative solutions may be used to solve the problem. Examining the suggested method's convergence, After the study's conclusion, we computed their soft L2 and soft L1 and gave a few numerical examples.

Keywords: Convergence analysis, fractional time Schrödinger equation, cumulative Jacobi spectral method, least Gaussian method

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تحليل ودراسة معادلات شرودنجر الكسرية للزمن

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المستخلص

في هذا البحث، تم استخدام طريقة جاكوبي للتماسك الطيفي الزمكاني (طريقة JSC) لحل معادلات شرودنغر التفاضلية غير الخطية الكسرية الزمنية وفقاً للشروط الأولية المناسبة. في البداية، يتم تحويل المشكلة المدروسة إلى نظام ذي صلة من المعادلات التفاضلية الجزئية ذات الفولتية التكاملية غير الخطية ذات النواة الضعيفة المفردة مع تعريف وخصائص عوامل المشتقة الكسرية والتكاملية. لذلك من خلال تكامل النظام المتعلق بالمعادلات التفاضلية التكاملية في المتغيرات المكانية والزمانية مع تقريب التكامل في المعادلة باستخدام صيغة جاكوبي غاوس التربيعية، يتم تقليص المشكلة إلى مجموعة من المعادلات الجبرية غير الخطية. يمكننا التفكير في حل النظام من خلال بعض الحلول التكرارية القوية. ومن أجل التحقق من تقارب الطريقة المقترحة قدمنا بعض الأمثلة العددية وحسبنا soft L^∞ و soft L^2 في نهاية البحث.

الكلمات المفتاحية: تحليل التقارب، معادلة شرودنغر للزمن الجزئي، طريقة جاكوبي الطيفية التراكمية، طريقة غاوس الصغرى

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1. INTRODUCTION

Schrödinger's equation plays an essential role in explaining the concepts of quantum physics. In fact, this equation provides an elegant expression of changes in physical systems (especially temperature changes) in quantum states. Schrödinger's equation in time fraction state is expressed as follows [1]:

$$i\frac{\partial^{\mu}\psi(x.y.t)}{\partial t^{\mu}} = a_1\frac{\partial^2\psi(x.y.t)}{\partial x^2} + a_2\frac{\partial^2\psi(x.y.t)}{\partial y^2} + \gamma|\psi(x.y.t)|^2\psi(x.y.t).$$

$$0 < \mu < 1. \quad (x.y.t) \in \Omega_1 \times \Omega_2 \times \Omega_3 \qquad (1)$$

These are the starting (time) circumstances:

$$\psi(x.y.0) = \zeta_1(x.y). \quad (x.y) \in \Omega_1 \times \Omega_2 \qquad (2)$$

And the (spatial) boundary conditions are considered as follows:

$$\psi(0.y.t) = \zeta_2(y.t). \quad \psi(L_1.y.t) = \zeta_3(y.t). \quad (y.t) \in \Omega_2 \times \Omega_3.$$
$$\psi(x.0.t) = \zeta_4(x.t). \quad \psi(x.L_2.t) = \zeta_5(x.t). \quad (x.t) \in \Omega_1 \times \Omega_3 \qquad (3)$$

In this article, we want to find an approximation of its solutions in terms of second-order Gaussian functions after rewriting Schrödinger's equation from the complex state to the real state, using derivative and fractional integral operators. First, we remind that the derivative of a fraction is defined as follows from Caputo's point of view:

$$\frac{\partial^\mu}{\partial\tau^\mu}\big(g(\tau)\big) = \begin{cases} \frac{\partial^m g(\tau)}{\partial\tau^m}. & \mu = m \in \mathbb{N}. \\ \frac{1}{\Gamma(m-\mu)}\int_0^\tau \frac{g^{(m)}(s)}{(\tau-s)^{\mu-m+1}}ds. & m-1 < \mu < m. \end{cases} \qquad (4)$$

And Riemann-Liouville's fractional integral is also defined as follows:

$$l_\tau^\mu\big(g(\tau)\big) = \frac{1}{\Gamma(\mu)}\int_0^\tau (\tau - s)^{\mu-1} g(s)ds. \qquad s > 0 \qquad (5)$$

Of course, there are other equivalent definitions. As we expect, these two definitions are somewhat opposite of each other. In fact, it is proved that:

$$l_\tau^\mu\left(\frac{\partial^\mu}{\partial\tau^\mu}g(\tau)\right) = g(\tau) - \sum_{i=0}^{m-1} g^{(i)}(0)\frac{\tau^i}{i!}. \quad m-1 < \mu < m \qquad (6)$$

So far, various methods have been proposed to investigate fractional differential equations. In differential and integral calculus, we face derivatives and integrals of arbitrary order (and not necessarily the correct order). Fractional derivatives have been used in a great deal of scientific and technical challenges in recent years. Numerous writers have reviewed applications of integral and differential calculus. For instance, the nonlinear oscillation model of an earthquake is studied using differential calculus and fractional integral.. Also, dynamic traffic models use this tool. They profit from these notions in the following areas: economics, signal processing, control theory, statistical chain mechanics, solid mechanics, oscillatory behavior of numerous elastic materials, and interactions between nanoparticles, among others[2].

The reason for the widespread use of differential calculus and fractional integral is that in nature, phenomena are not only dependent on the present time, but also on previous moments, and the equations including fractional derivative and integral provide a better expression of them[3].

The mathematical formulation of many of the mentioned phenomena includes non-linear integral equations in the fractional state. Most of these equations do not have exact analytical solutions, so numerical and approximate methods are suggested to find their approximate solutions[4].

In this article, we examine a group of non-linear fractional integral equations of the Schrödinger type and provide appropriate numerical methods for them. The existence of solutions, both local

and general, as well as unique solutions of integral equations with given initial conditions, have been investigated and presented.

There are various methods for solving integral equations with fractional derivative, such as change of variable, decomposition method, iterative method, homotopy perturbation method, wavelet method, and Taylor expansion method[4].

The wave approach appears to be the most appealing and versatile of these techniques. The wave theory has been applied extensively in many scientific and engineering domains in recent years. To put it simply, wavelet uses quick numerical techniques to accurately represent a variety of functions, operators, and their relationships [5].

Polynomial series and orthogonal functions are particularly effective in this sense. This method's primary benefit is that it makes it easier to solve systems of algebraic equations. As a result, there is much room for solution to these issues. Numerous wavelet types, including the Legendre wavelet, Legendre multiple wavelet, Harr wavelet, Chebyshev wavelet, and others, are frequently employed in the numerical solution of non-linear integral equations, differential equations, and integral equations among other types of equations[6].

It should be noted that every equation in the list is of integer order. Numerical solutions based on wavelet theory have been offered only in a small number of particular circumstances for integral equations with erroneous order. Often, the author proposes a wavelet-based numerical method for solving fractional integral equations and compares its numerical solution with its exact solution [7].

In short, due to the lack of a complete and comprehensive analytical solution for such equations, often and inevitably, appropriate numerical and approximate solutions are suggested to estimate their solutions, which this article, from the perspective of Jacobi and functions Gaussian is used for this purpose. Of course, the appropriate solution of the resulting non-linear device is also a significant issue, and here, we use appropriate iterative methods to solve it [8].

Fractional derivative and fractional integral operators are among the newest and most important tools in mathematical analysis and numerical analysis. In this thesis, we first introduce Caputo's fractional derivative and Riemann-Liouville's fractional integral and examine their basic properties [9].

Then, with the help of fractional derivative and fractional integral operators, we present a simple calculation method for solving and checking a group of fractional differentials and integral equations [10].

Also, we calculate the operational matrix corresponding to the fractional derivative and fractional integral operators and use it to convert the desired equation into a device of non-linear equations[10].

In the end, we test the presented method based on applications and numerical results and compare it with previous methods. This comparison shows that the method presented in this article is more effective and more accurate than the previous methods [11].

Using Jacobi time-space perspective can effectively improve the accuracy and speed of calculations. Using this perspective, we first divide all of the functions in the equation—known and unknown—into their real and imaginary components.. Then, we rewrite the desired equation as a differential equation with partial derivatives from Caputo's point of view. In the following, we apply Riemann and Liouville integral operators on both sides of the equation.

With this work, the desired equation becomes a system of partial differential equations of the Volterra type with weak kernels and initial conditions[12]. Finally, the desired issue is reduced to a set of non-linear equations by integrating the temporal and spatial variables together and estimating the integrals that already exist with the use of Gaussian quadratic laws. For example, Newton's method and appropriate iterative techniques can be used to solve the resultant non-linear device.

2. RELATED WORK

In the theory of quantum mechanics, Schrödinger's equation represents a major advancement[12]. This equation gives a strong differential structure associated with quantum system changes when quantum influences become significant.

It should be noted that the first order and correct order spatial partial derivatives are used to represent this equation in its classical form. In practical applications, the fractional kind of Schrödinger equation can more accurately depict physical and chemical phenomena because fractional differential operators are non-local[13]. [14] presented the idea of Schrödinger's fractional equation, which extends the Feyman route integral. Schrödinger's fractional equations are nonlinear, complicated, and nonlocal, making classical approaches ineffective for solving PDEs. However, it is crucial for researchers to solve these equations in order to have predictable behavior and accurately simulate occurrences. Thus, in order to effectively compute the solution of fractional Schrödinger equations, numerical and analytical approaches are to be investigated and extended [15].

Scientists have put forth a number of analytical and numerical techniques in recent years to solve Schrödinger's fractional equation. Homotopy analysis (HAM) is one analytical technique that might be considered [6]. Any nonlinear PDE may be solved analytically using extremely basic techniques that don't include any discretization or linearization steps. However, these approaches have the drawback of being time-consuming. Because these methods use symbols to carry out the integration and differentiation processes. In the meanwhile, two effective tools in numerical techniques that accelerate (and shorten computing time) the differentiation and integration processes are Gaussian quadrature rules and differentiation operational matrices.. Additionally, since the solutions to these equations and schemes depend on the right-hand operator functions and maintain their behavior, software-free solutions can be computed in some numerical approaches, such as Krylov subspace methods, without the need for regularization tools[7]..

Consequently, researchers find it more appealing to use numerical approaches. Low-order numerical approaches, such as kernel reproduction methods [13]. and local discontinuous Galerkin (DG) [11], might be mentioned among the numerical techniques.. Recently, the idea of solving the Dinger and Schrödinger equations in a fraction of the time was put out. Note that, because fractional differential operators are global, it is best to solve the studied equations with specific global numerical methods. Considering that, to solve a partial differential equation (PDE), the time variable is discretized using global techniques like the radial basis function harmonic technique, In order to tackle nonlinear fractional parabolic time problems, spatial variables are localized using local methods such Galerkin Finite Element Methods (FEMs) [14]. [13] the application of

finite difference techniques (FDM) can lead to an unbalanced numerical scheme with spectral accuracy in the space variable and low-order algebraic precision in the time variable. Thus, a balanced numerical technique with spectrum precision in both temporal and spatial dimensions is desirable to suggest. Spectral Galerkin techniques make it simple to implement fractional differential equations (FDEs), particularly when solving nonlinear FDEs. These techniques have shown effective in solving integral equations (IEs) via accurate convergence analysis, fractional nonlinear boundary value problems (BVPs) [9], and FDEs with non-smooth regularized solutions.. Furthermore, Schrödinger's fractional equations were only solved numerically by the author in[7]. via the spectral integration approach described in [12]. Additionally, to solve the nonlinear fractional time multidimensional Schrödinger equation, implicit alternating direction (ADI) methods and a linearized L 1-Galerkin finite element approach are presented, respectively. To the best of the authors' knowledge, nonlinear time-fractional Schrödinger equations have not been solved much using Jacobian space-time spectral coherence approaches (backed by an accurate convergence analysis). Our conclusion is that a balanced approach to solving the fractional-time Schrödinger equations is what we propose: a Jacobian spectral alignment system with a comprehensive convergence analysis and spectral accuracy in both temporal and spatial dimensions.

3. JACOBI'S METHOD FOR CHECKING DIFFERENTIAL EQUATIONS

Calculation accuracy and speed can be effectively increased by using the Jacobi time-space viewpoint. First, we divide all of the functions in the equation—known and unknown—into their real and imaginary components according to this viewpoint. Next, using Caputo's perspective, we recast the desired problem as a differential equation with partial derivatives. Riemann and Liouville integral operators are applied on both sides of the equation in the following. This approach transforms the desired problem into a Volterra-type system of partial differential equations with weak kernels and starting conditions.. Finally, the desired issue is reduced to a set of nonlinear equations by integrating the temporal and spatial variables together and estimating the integrals that already exist with the use of Gaussian quadratic laws. The resultant nonlinear device can be solved, for instance, using appropriate iterative techniques and Newton's method.

We now go over these procedures in further depth. First, we divide each function in the Schrödinger equation into its real and imaginary components, including both known and unknown functions. In terms of mathematics, we have:

$$\boldsymbol{\psi = u + iv.} \quad \boldsymbol{R = f + ig.} \quad \boldsymbol{\zeta_1 = g_1 + ig_2.}$$

$$\boldsymbol{\zeta_2 = g_3 + ig_4.} \quad \boldsymbol{\zeta_3 = g_5 + ig_6.}$$

$$\boldsymbol{\zeta_4 = g_7 + ig_8.} \quad \boldsymbol{\zeta_5 = g_9 + ig_{10}.} \qquad (7)$$

With this rewriting, the main equation becomes as follows:

$$i\left(\frac{\partial^{\mu}u}{\partial t^{\mu}}-a_1\frac{\partial^2 v}{\partial x^2}-a_2\frac{\partial^2 v}{\partial y^2}-\gamma(u^2+v^2)v-\delta g\right)-\left(\frac{\partial^{\mu}v}{\partial t^{\mu}}+a_1\frac{\partial^2 u}{\partial x^2}+a_2\frac{\partial^2 u}{\partial y^2}-\gamma(u^2+v^2)u+\delta f\right)=0 \quad (8)$$

$$\frac{\partial^{\mu}u}{\partial t^{\mu}}=a_1\frac{\partial^2 v}{\partial x^2}+a_2\frac{\partial^2 v}{\partial y^2}+\gamma(u^2+v^2)v+\delta g. \quad (9)$$

$$-\frac{\partial^{\mu}v}{\partial t^{\mu}}=a_1\frac{\partial^2 u}{\partial x^2}+a_2\frac{\partial^2 u}{\partial y^2}+\gamma(u^2+v^2)u\delta f \quad (10)$$

which are considered under the following basic conditions:

$$u(x.y.0)=g_1(x.y).\ \ v(x.y.0)=g_2(x.y).\ \ (x.y)\in\Omega_1\times\Omega_2. \quad (11)$$

$$u(0.y.t)=g_3(y.t).\ \ v(0.y.t)=g_4(y.t). \quad (12)$$

$$u(L_1.y.t)=g_5(y.t).\ \ v(L_1.y.t)=g_6(y.t).\ \ (y.t)\in\Omega_2\times\Omega_3. \quad (13)$$

$$u(x.0.t)=g_7(x.t).\ \ v(x.0.t)=g_8(x.t). \quad (14)$$

$$u(x.L_2.t)=g_9(x.t).\ \ v(x.L_2.t)=g_{10}(y.t).\ \ (x.t)\in\Omega_1\times\Omega_3. \quad (15)$$

Considering this issue, equations (9)- (10) and (11)- (15) are equal to equations (2) and (3). Therefore, instead of the numerical solution of (2) and (3), we calculate the numerical solution of (9)-(10) and (11)- (15) using the Jacobian spectral interpolation method. Before the space-time coherence method, we first convert the equations (9)-(10) into the system related to Volterra integral differential equations with a weak single kernel by using the Riemann and Liouville fractional integrals of μ order. In the following, we apply Riemann and Liouville integral operators on both sides of the equation. We remind you that this integral operator is defined as follows:

$$u(x.y.t)=\frac{1}{\Gamma(\mu)}\int_0^t(t-\tau)^{\mu-1}k_1\big(x.y.\tau.u(x.y.\tau).v(x.y.\tau)\big)d\tau+\delta\tilde{g}(x.y.t)+g_1(x.y). \quad (16)$$

$$v(x.y.t)=\frac{1}{\Gamma(\mu)}\int_0^t(t-\tau)^{\mu-1}k_2\big(x.y.\tau.u(x.y.\tau).v(x.y.\tau)\big)d\tau+\delta\tilde{f}(x.y.t)+g_2(x.y). \quad (17)$$

where nuclear functions are defined as follows:

$$k_1\big(x.y.\tau.u(x.y.\tau).v(x.y.\tau)\big)=a_1\frac{\partial^2 v(x.y.t)}{\partial x^2}+a_2\frac{\partial^2 v(x.y.t)}{\partial y^2}+\gamma\left(u^2(x.y.\tau)+v^2(x.y.\tau)\right)v(x.y.\tau). \quad (18)$$

$$k_2\big(x.y.\tau.u(x.y.\tau).v(x.y.\tau)\big) = -a_1\frac{\partial^2 u(x.y.\tau)}{\partial x^2} - a_2\frac{\partial^2 u(x.y.t)}{\partial y^2} - \gamma\Big(u^2(x.y.\tau)+v^2(x.y.\tau)\Big)u(x.y.\tau) \quad (19)$$

$$\tilde{g}(x.y.t) = \frac{1}{\Gamma(\mu)}\int_0^t (t-\tau)^{\mu-1} g(x.y.\tau)d\tau. \quad (20)$$

$$\tilde{f}(x.y.t) = -\frac{1}{\Gamma(\mu)}\int_0^t (t-\tau)^{\mu-1} f(x.y.\tau)d\tau. \quad (21)$$

Implementing the numerical procedure (16)-(17) might be challenging because of the weak kernels around + t = 0. Consider the following variable changes in order to use orthogonal Jacobi polynomials to solve equations (16)–(17).

$$x = \frac{L_1}{2}(1+\bar{x}).\ \ \bar{x} = \frac{2x}{L_1} - 1.\ \ \ \bar{x} \in [-1.1]. \quad (22)$$

$$y = \frac{L_2}{2}(1+\bar{y}).\ \ \bar{y} = \frac{2y}{L_2} - 1.\ \ \ \bar{y} \in [-1.1]. \quad (23)$$

$$t = \frac{T}{2}(1+\bar{t}).\ \ \bar{t} = \frac{2t}{T} - 1.\ \ \ t \in [-1.1]. \quad (24)$$

$$\tau = \frac{T}{2}(1+s).\ \ s = \frac{2\tau}{T} - 1.\ \ \ s \in [-1.1]. \quad (25)$$

Therefore, equations (15)-(16) should be rewritten in the following form:

$$\bar{u}(x.y.t) = \int_{-1}^t (t-s)^{\mu-1}\bar{k}_1\big(x.y.s.\bar{u}(x.y.s).\bar{v}(x.y.s)\big)ds + \delta\tilde{g}(x.y.t) + \bar{g}_1(x.y). \quad (26)$$

$$\bar{v}(x.y.t) = \int_{-1}^t (t-s)^{\mu-1}\bar{k}_2\big(x.y.s.\bar{u}(x.y.s).\bar{v}(x.y.s)\big)ds + \delta\tilde{f}(x.y.t) + \bar{g}_2(x.y). \quad (27)$$

which are considered under the following boundary conditions:

$$\bar{u}(-1.y.t) = \bar{g}_3(y.t).\ \ \bar{u}(1.y.t) = \bar{g}_5(y.t). \quad (28)$$

$$\bar{u}(x.-1.t) = \bar{g}_7(x.t).\ \ \bar{u}(x.1.t) = \bar{g}_9(y.t). \quad (29)$$

$$\bar{v}(-1.y.t) = \bar{g}_4(y.t).\ \ \bar{v}(1.y.t) = \bar{g}_6(y.t). \quad (30)$$

$$\bar{v}(x.-1.t) = \bar{g}_8(x.t).\ \ \bar{v}(x.1.t) = \bar{g}_{10}(y.t). \quad (31)$$

That

$$\bar{u}(x.y.t) = u\left(\frac{L_1}{2}(1+x).\ \frac{L_2}{2}(1+y).\frac{T}{2}(1+t)\right) \quad (32)$$

$$\bar{v}(x.y.t) = v\left(\frac{L_1}{2}(1+x).\ \frac{L_2}{2}(1+y).\frac{T}{2}(1+t)\right) \quad (33)$$

$$\bar{g}(x.y.t) = \tilde{g}\left(\frac{L_1}{2}(1+x).\ \frac{L_2}{2}(1+y).\frac{T}{2}(1+t)\right) \quad (34)$$

$$\bar{f}(x.y.t) = \tilde{f}\left(\frac{L_1}{2}(1+x).\ \frac{L_2}{2}(1+y).\frac{T}{2}(1+t)\right) \quad (35)$$

$$\bar{g}_i(x.y) = g_i\left(\frac{L_1}{2}(1+x).\frac{L_2}{2}(1+y)\right).\ i = 1.2. \quad (36)$$

$$\bar{g}_i(y.t) = g_i\left(\frac{L_2}{2}(1+y).\frac{T_2}{2}(1+t)\right).\ i = 3.4.5.6 \quad (37)$$

And

$$\bar{g}_i(x.t) = g_i\left(\frac{L_1}{2}(1+x).\frac{T_2}{2}(1+t)\right).\ i = 7.8.9.10. \quad (38)$$

$$\bar{k}_1\left(x.y.s.\bar{u}(x.y.s).\bar{v}(x.y.s)\right) = \frac{1}{\Gamma(\mu)}\left(\frac{T}{2}\right)^{\mu}\left(a_1\left(\frac{2}{L_1}\right)^2\frac{\partial^2\bar{v}}{\partial x^2} + a_2\left(\frac{2}{L_2}\right)^2\frac{\partial^2\bar{v}}{\partial y^2} + \gamma\left(\bar{u}^2+\bar{v}^2\right)\bar{v}\right). \quad (39)$$

$$\bar{k}_2\left(x.y.s.\bar{u}(x.y.s).\bar{v}(x.y.s)\right) = \frac{1}{\Gamma(\mu)}\left(\frac{T}{2}\right)^{\mu}\left(-a_1\left(\frac{2}{L_1}\right)^2\frac{\partial^2\bar{u}}{\partial x^2} - a_2\left(\frac{2}{L_2}\right)^2\frac{\partial^2\bar{u}}{\partial y^2} - \gamma\left(\bar{u}^2+\bar{v}^2\right)\bar{u}\right). \quad (40)$$

Now we can put the variable t of equations (22)-(25) in the Gauss Jacobi nodes corresponding to θ=ϑ=-μ, where:

$$\omega^{\theta\cdot\vartheta}(t) = (1-t)^{-\mu}(1+t)^{-\mu} = \left(1-t^2\right)^{-\mu}.\quad 0 \le l \le M \quad (41)$$

$$\bar{u}(x.y.t_l) = \int_{-1}^{t_l}(t_l-s)^{\mu-1}\bar{k}_1\left(x.y.s.\bar{u}(x.y.s).\bar{v}(x.y.s)\right)ds + \delta\tilde{g}(x.y.t_l) + \bar{g}_1(x.y). \quad (42)$$

$$\bar{v}(x.y.t_l) = \int_{-1}^{t_1}(t_l-s)^{\mu-1}\bar{k}_2\left(x.y.s.\bar{u}(x.y.s).\bar{v}(x.y.s)\right)ds + \delta\tilde{f}(x.y.t_l) + \bar{g}_2(x.y). \quad (43)$$

At this stage, to implement Jacobi Gauss' quadrature law, we must change the following variables:

$$s(\theta) = s_1(\theta) = \frac{1+t_l}{2}\theta + \frac{t_l-1}{2}.\quad 0 \le l \le M.\quad \theta \in [-1.1] \quad (44)$$

So

$$\bar{u}(x.y.t_l) = \left(\frac{1+t_1}{2}\right)^{\mu}\int_{-1}^{1}(1-\theta)^{\mu-1}\bar{k}_1\left(x.y.s(\theta).\bar{u}\left(x.y.s(\theta)\right).\bar{v}\left(x.y.s(\theta)\right)\right)d\theta + \delta\bar{g}(x.y.t_l) + \bar{g}_1(x.y) \quad (45)$$

$$\bar{v}(x.y.t_l) = \left(\frac{1+t_1}{2}\right)^{\mu} \int_{-1}^{1} (1-\theta)^{\mu-1} \bar{k}_2 \left(x.y.s(\theta).\bar{u}(x.y.s(\theta)).\bar{v}(x.y.s(\theta))\right) d\theta$$

$$+\delta \bar{f}(z.y.t_l) + \bar{g}_2(x.y) \quad (46)$$

By using the Gaussian quadratic formula to change the integral components of the above formulas collectively, we conclude that:

$$\int_{-1}^{1} (1-\theta)^{\mu-1} \bar{k}_1 \left(x.y.s(\theta).\bar{u}(x.y.s(\theta)).\bar{v}(x.y.s(\theta))\right) d\theta \approx$$

$$\sum_{k=0}^{L} \bar{k}_1 \left(x.y.s(\theta_k).\bar{u}(x.y.s(\theta_k)).\bar{v}(x.y.s(\theta_k))\right) \omega_k^{\mu-1.0}. \quad (47)$$

$$\int_{-1}^{1} (1-\theta)^{\mu-1} \bar{k}_2 \left(x.y.s(\theta).\bar{u}(x.y.s(\theta)).\bar{v}(x.y.s(\theta))\right) d\theta \approx$$

$$\sum_{k=0}^{L} \bar{k}_2 \left(x.y.s(\theta_k).\bar{u}(x.y.s(\theta_k)).\bar{v}(x.y.s(\theta_k))\right) \omega_k^{\mu-1.0}. \quad (48)$$

where $\{\theta_k\}_{k=0}^{L}$ represent Jacobi points of Gauss and Lobato and $\left\{\omega_k^{\mu-1.0}\right\}_{k=0}^{L}$ are weight functions in the interval [- 1.1] are and L⩾M. These weight functions are defined as follows:

$$\omega^{\mu-1.0}(t) = (1-t)^{\mu-1} \quad (49)$$

Suppose $u^{-l}(x.y)$ and $v^{-l}(x.y)$ represent the symbols $\bar{u}(x.y.t_l)$ and $\bar{v}(x.y.t_l)$, respectively. At this stage, we can approximate $\bar{u}(x.y.t)$ and $\bar{v}(x.y.t)$ with the help of the corresponding Lagrange polynomials as follows:

$$\bar{u}(x.y.t) \approx u^{-M}(x.y.t) = \sum_{l=0}^{M} u^{-l}(x.y) F_l(t). \quad (50)$$

$$\bar{v}(x.y.t) \approx v^{-M}(x.y.t) = \sum_{l=0}^{M} v^{-l}(x.y) F_l(t). \quad (51)$$

It is possible to reduce (32)-(36) by using the Gauss quadrature law and the previously provided approximations :

$$\bar{u}^l(x.y) = \left(\frac{1+t_1}{2}\right)^{\mu} \sum_{k=0}^{L} \bar{k}_1 \left(x.y.s(\theta_k).\bar{u}^M(x.y.s(\theta_k)).\ \bar{v}^M(x.y.s(\theta_k))\right) \omega_k^{\mu-1.0} + \delta \bar{g}(x.y.t_l) + \bar{g}_1(x.y). \quad (52)$$

$$\bar{v}^l(x.y) = \left(\frac{1+t_1}{2}\right)^{\mu} \sum_{k=0}^{L} \bar{k}_2 \left(x.y.s(\theta_k).\bar{u}^M(x.y.s(\theta_k)).\ \bar{v}^M(x.y.s(\theta_k))\right) \omega_k^{\mu-1.0} + \delta \bar{f}(x.y.t_l) + \bar{g}_2(x.y). \quad (53)$$

To apply the JSC method on the variable space, we use Legendre Gauss Lobatto points in the form $\{x_i\}_{i=0}^{N_1}$ and $\{y_j\}_{j=0}^{N_2}$ which correspond to The following weighting functions are:

$$\boldsymbol{\omega^{0.0}(x.y) = (1-x)^0(1+x)^0(1-y)^0(1+y)^0 = 1} \qquad \textbf{(54)}$$

With this work, the relations (40)-(42) becomes as follows:

$$\bar{\boldsymbol{u}}^{\boldsymbol{j}}(\boldsymbol{x_i.y_j}) = \left(\frac{\boldsymbol{1+t_l}}{\boldsymbol{2}}\right)^{\boldsymbol{\mu}} \sum_{\boldsymbol{k=0}}^{\boldsymbol{L}} \bar{\boldsymbol{k}}_{\boldsymbol{1}}\left(\boldsymbol{x_i.y_j.s(\theta_k)}.\bar{\boldsymbol{u}}^{\boldsymbol{M}}\left(\boldsymbol{x_i.y_j.s(\theta_k)}\right).\ \bar{\boldsymbol{v}}^{\boldsymbol{M}}\left(\boldsymbol{x_i.y_j.s(\theta_k)}\right)\right)\boldsymbol{\omega_k^{\mu-1.0}}$$

$$+\boldsymbol{\delta}\bar{\boldsymbol{g}}(\boldsymbol{x_i.\ y_j.t_l}) + \bar{\boldsymbol{g}}_{\boldsymbol{1}}(\boldsymbol{x_i.\ y_j}). \qquad \textbf{(55)}$$

$$\bar{\boldsymbol{v}}^{\boldsymbol{j}}(\boldsymbol{x_i.y_j}) = \left(\frac{\boldsymbol{1+t_l}}{\boldsymbol{2}}\right)^{\boldsymbol{\mu}} \sum_{\boldsymbol{k=0}}^{\boldsymbol{L}} \bar{\boldsymbol{k}}_{\boldsymbol{2}}\left(\boldsymbol{x_i.y_j.s(\theta_k)}.\bar{\boldsymbol{u}}^{\boldsymbol{M}}\left(\boldsymbol{x_i.y_j.s(\theta_k)}\right).\ \bar{\boldsymbol{v}}^{\boldsymbol{M}}\left(\boldsymbol{x_i.y_j.s(\theta_k)}\right)\right)\boldsymbol{\omega_k^{\mu-1.0}}$$

$$+\boldsymbol{\delta}\bar{\boldsymbol{f}}(\boldsymbol{x_i.\ y_j.t_j}) + \bar{\boldsymbol{g}}_{\boldsymbol{2}}(\boldsymbol{x_i.\ y_j}). \qquad \textbf{(56)}$$

Again, we use the symbols $\bar{u}_{ij}^{l}$ and $\bar{v}_{ij}^{l}$ to denote $\bar{u}(x_i.y_j.t_l)$ and $\bar{v}(x_i.y_j.t_l)$, respectively. We can write the Lagrange polynomial approximation for both of these functions:

$$\backslash\bar{\boldsymbol{u}}(\boldsymbol{x.y.t}) \approx \bar{\boldsymbol{u}}_{\boldsymbol{N_1N_2}}^{\boldsymbol{M}}(\boldsymbol{x.y.t}) = \sum_{\boldsymbol{l=0}}^{\boldsymbol{M}}\sum_{\boldsymbol{i=0}}^{\boldsymbol{N_1}}\sum_{\boldsymbol{j=0}}^{\boldsymbol{N_2}}\bar{\boldsymbol{u}}_{\boldsymbol{ij}}^{\boldsymbol{l}}\boldsymbol{H_i(x)H_j(y)F_l(t)} =$$

$$\sum_{\boldsymbol{l=0}}^{\boldsymbol{M}}\sum_{\boldsymbol{i=0}}^{\boldsymbol{N_1-1}}\sum_{\boldsymbol{j=0}}^{\boldsymbol{N_2-1}}\bar{\boldsymbol{u}}_{\boldsymbol{ij}}^{\boldsymbol{l}}\boldsymbol{H_i(x)H_j(y)F_l(t)} + \sum_{\boldsymbol{l=0}}^{\boldsymbol{M}}\sum_{\boldsymbol{j=0}}^{\boldsymbol{N_2}}\left(\bar{\boldsymbol{g}}_{\boldsymbol{3}}\boldsymbol{H_0(x)} + \bar{\boldsymbol{g}}_{\boldsymbol{4}}\boldsymbol{H_{N_1}(x)}\right)\boldsymbol{H_j(y)F_l(t)} +$$

$$\sum_{\boldsymbol{l=0}}^{\boldsymbol{M}}\sum_{\boldsymbol{i=0}}^{\boldsymbol{N_1}}\left(\bar{\boldsymbol{g}}_{\boldsymbol{7}}\boldsymbol{H_0(y)} + \bar{\boldsymbol{g}}_{\boldsymbol{8}}\boldsymbol{H_{N_2}(y)}\right)\boldsymbol{H_i(x)F_l(t)}\,\bar{\boldsymbol{v}}(\boldsymbol{x.y.t}) \approx \bar{\boldsymbol{v}}_{\boldsymbol{N_1N_2}}^{\boldsymbol{M}}(\boldsymbol{x.y.t}) =$$

$$\sum_{\boldsymbol{l=0}}^{\boldsymbol{M}}\sum_{\boldsymbol{i=0}}^{\boldsymbol{N_1}}\sum_{\boldsymbol{j=0}}^{\boldsymbol{N_2}}\bar{\boldsymbol{v}}_{\boldsymbol{ij}}^{\boldsymbol{l}}\boldsymbol{H_i(x)H_j(y)F_l(t)} = \sum_{\boldsymbol{l=0}}^{\boldsymbol{M}}\sum_{\boldsymbol{i=0}}^{\boldsymbol{N_1-1}}\sum_{\boldsymbol{j=0}}^{\boldsymbol{N_2-1}}\bar{\boldsymbol{v}}_{\boldsymbol{ij}}^{\boldsymbol{l}}\boldsymbol{H_i(x)H_j(y)F_l(t)} + \sum_{\boldsymbol{l=0}}^{\boldsymbol{M}}\sum_{\boldsymbol{j=0}}^{\boldsymbol{N_2}}\left(\bar{\boldsymbol{g}}_{\boldsymbol{5}}\boldsymbol{H_0(x)} +\right.$$

$$\left.\bar{\boldsymbol{g}}_{\boldsymbol{6}}\boldsymbol{H_{N_1}(x)}\right)\boldsymbol{H_j(y)F_l(t)} + \sum_{\boldsymbol{l=0}}^{\boldsymbol{M}}\sum_{\boldsymbol{i=0}}^{\boldsymbol{N_1}}\left(\bar{\boldsymbol{g}}_{\boldsymbol{9}}\boldsymbol{H_0(y)} + \bar{\boldsymbol{g}}_{\boldsymbol{10}}\boldsymbol{H_{N_2}(y)}\right)\boldsymbol{H_i(x)F_l(t)} \qquad \textbf{(57)}$$

And the Lagrange polynomials for x_i and y_i are represented by the variables H_i (x) and H_j (y), respectively. In order to solve (1) and (2), the entire discrete system of algebraic equations resulting from the space-time spectral localization approach can be written as Ultimately, the relationships that arise result in a system of non-linear equations that look like this (all the required equations may be achieved by varying the i and j variables :

$$\bar{\boldsymbol{u}}_{\boldsymbol{ij}}^{\boldsymbol{l}} = \left(\frac{\boldsymbol{1+t_1}}{\boldsymbol{2}}\right)^{\boldsymbol{\mu}} \sum_{\boldsymbol{k=0}}^{\boldsymbol{L}} \bar{\boldsymbol{k}}_{\boldsymbol{1}}\left(\boldsymbol{x_i.y_j.s(\theta_k)}.\bar{\boldsymbol{u}}_{\boldsymbol{N_1N_2}}^{\boldsymbol{M}}\left(\boldsymbol{x_i.y_j.s(\theta_k)}\right).\bar{\boldsymbol{v}}_{\boldsymbol{N_1N_2}}^{\boldsymbol{M}}\left(\boldsymbol{x_i.y_j.s(\theta_k)}\right)\right)\boldsymbol{\omega_k^{\mu-1.0}} + \boldsymbol{\delta}\bar{\boldsymbol{g}}(\boldsymbol{x_i.\ y_j.t_l})$$

$$+ \bar{\boldsymbol{g}}_{\boldsymbol{1}}(\boldsymbol{x_i.\ y_j}). \qquad \textbf{(58)}$$

$$= \left(\frac{1+t_1}{2}\right)^{\mu} \sum_{k=0}^{L} \bar{k}_1(x_i.y_j.s(\theta_k). \sum_{n_1=0}^{N_1}\sum_{n_2=0}^{N_2}\sum_{m=0}^{M} \bar{u}_{n_1n_2}^{m} H_{n_1}(x_i)H_{n_2}(y_j)F_m(s(\theta_k)).$$

$$\sum_{n_1=0}^{N_1}\sum_{n_2=0}^{N_2}\sum_{m=0}^{M} \bar{v}_{n_1n_2}^{m} H_{n_1}(x_i)H_{n_2}(y_j)F_m(s(\theta_k)))\omega_k^{\mu-1.0}$$

$$+\delta\bar{g}(x_i.\ y_j.t_l) + \bar{g}_1(x_i.\ y_j). \qquad (59)$$

$$\bar{v}_{ij}^{l} = \left(\frac{1+t_1}{2}\right)^{\mu}$$

$$\sum_{k=0}^{L} \bar{k}_1\left(x_i.y_j.s(\theta_k).\bar{u}_{N_1N_2}^{M}\left(x_i.y_j.s(\theta_k)\right).\bar{v}_{N_1N_2}^{M}\left(x_i.y_j.s(\theta_k)\right)\right)\omega_k^{\mu-1.0}$$

$$+\delta\bar{f}(x_i.\ y_j.t_l) + \bar{g}_2(x_i.\ y_j). \qquad (60)$$

$$= \left(\frac{1+t_1}{2}\right)^{\mu}$$

$$\sum_{k=0}^{L} \bar{k}_1(x_i.y_j.s(\theta_k). \sum_{n_1=0}^{N_1}\sum_{n_2=0}^{N_2}\sum_{m=0}^{M} \bar{u}_{n_1n_2}^{m} H_{n_1}(x_i)H_{n_2}(y_j)F_m(s(\theta_k)).$$

$$\sum_{n_1=0}^{N_1}\sum_{n_2=0}^{N_2}\sum_{m=0}^{M} \bar{v}_{n_1n_2}^{m} H_{n_1}(x_i)H_{n_2}(y_j)F_m(s(\theta_k)))\omega_k^{\mu-1.0}$$

$$+\delta\bar{f}(x_i.\ y_j.t_l) + \bar{g}_2(x_i.\ y_j). \qquad (61)$$

We can solve this non-linear device with appropriate iterative methods and find the answer with desired accuracy. It is possible to answer this question by using Newton-Raphson's iterative approach to solve the nonlinear algebraic equations in the system mentioned above. For instance, MATLAB or Maple software's well-known "fsolve" tool can be used to carry out the actual implementation.

4. IMPLEMENTATION

4.1 Numerical examples

Example 1. 1-dimensional differential equation.

We consider the following one-dimensional differential equation (considered under the domain $(x.t) \in (-1.1) \times (0.2)$:

$$i\frac{\partial^{3/4}}{\partial t^{3/4}}\psi(x.t)+\frac{\partial^2}{\partial x^2}\psi(x.t)=f_1(x.t).\quad (x.t)\in(-1.1)\times(0.2)$$

$$\psi(x.0)=0.\quad x\in(-1.1)$$

$$\psi(-1.t)=\psi(1.t)=-t^2.\quad t\in(0.2) \qquad (62)$$

That

$$f_1=\frac{16\sqrt{2}}{5\pi}\Gamma\left(\frac{3}{4}\right)t^{5/4}(i\cos\pi x-\sin\pi x)+t^2\left(-\pi^2\cos\pi x-i\pi^2\sin\pi x\right) \qquad (63)$$

The exact answer to this equation is as follows:

$$\psi(x.t)=t^2(\cos\pi x+i\sin\pi x)$$

As we see in the figure (1), also we see result in table (1).

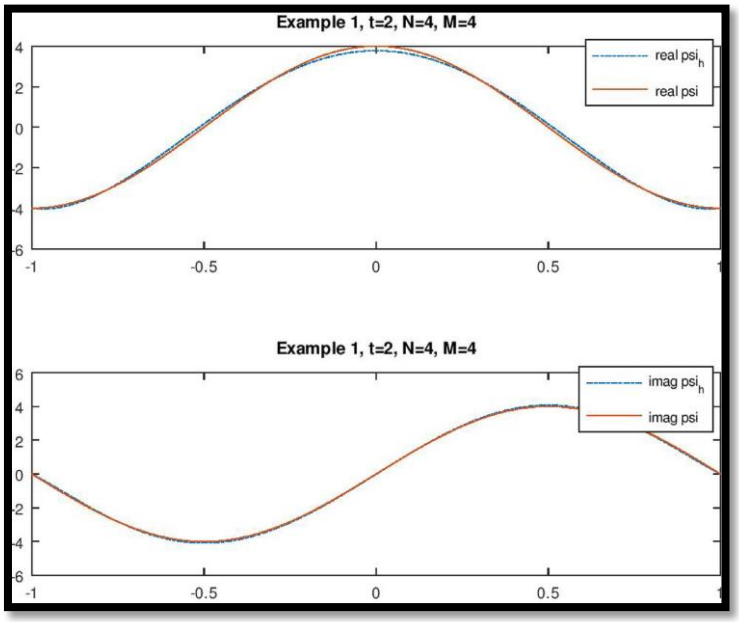

Figure (1): - Results related to example 1

Table (1): Checking the error values related to example 1.

M, N	$\|\psi-\psi_h\|_{L^\infty}$	$\|\psi-\psi_h\|_{L^2}$
4, 4	2.2628E-001	6.3147E-002
6, 6	9.1637E-003	1.2706E-003
8, 8	2.5137E-004	2.1386E-005
10, 10	5.4994E-006	1.8660E-006
12, 12	8.5810E-007	8.3305E-007

Example 2. One dimensional differential equation

We consider the following differential equation:

$$i\frac{\partial^{7/8}}{\partial t^{7/8}}\psi+\frac{\partial^2}{\partial x^2}\psi+|\psi|^2\psi=f_2.\quad (x.t)\in(-1.1)\times(0.2)$$

$$\psi(x.0)=0.\quad x\in(-1.1)$$

$$\psi(-1.t)=\psi(1.t)=0.\quad t\in(0.2)$$

The exact answer for f_2 is as follows:

$$\boldsymbol{\psi(x.t)=t^{15/8}\left(\frac{1}{8}\sin 2\pi x+\frac{i}{12}\sin \pi x\right)}$$

(64)

As we see in the figure (2), also we see result in table (2).

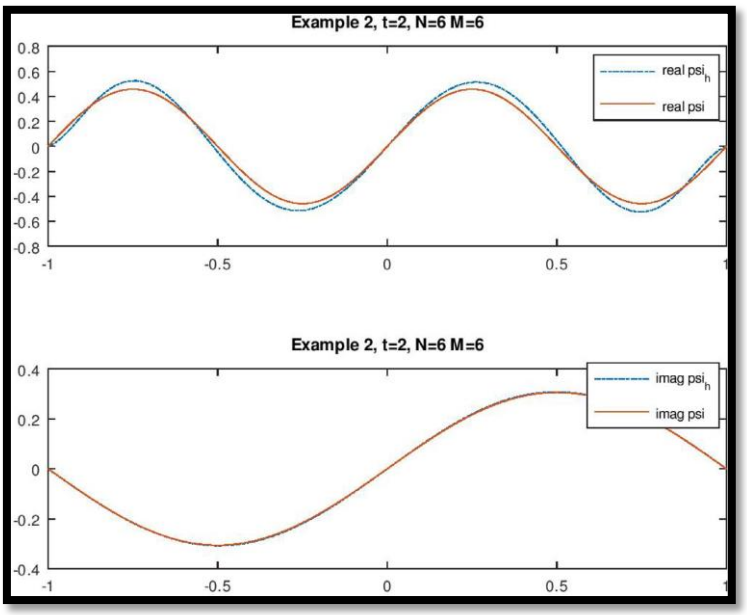

Figure (2): Results related to example 2

Table (2): Checking the error values related to example 2.

M, N	$\|\psi-\psi_h\|_{L^\infty}$	$\|\psi-\psi_h\|_{L^2}$
4, 4	8.1372E-001	5.6689E-001
6, 6	7.3603E-002	4.1019E-002
8, 8	6.6883E-003	2.7107E-003
10, 10	4.7876E-004	1.4701E-004
12, 12	2.7144E-005	1.2426E-005
14, 14	5.0659E-006	5.1181E-006

Example 3: Two-dimensional differential equation.

We consider the following Schrödinger differential equation:

$$\boldsymbol{i\frac{\partial^{7/8}}{\partial t^{7/8}}\psi+\frac{\partial^2}{\partial x^2}\psi+\frac{\partial^2}{\partial y^2}\psi+|\psi|^2\psi=f_3.\quad (x.y.t)\in(-1.1)^2\times(0.2)}\qquad \textbf{(64)}$$

$$\psi(x.y.0)=0.\quad (x.y)\in(-1.1)^2$$

$$\psi(\pm1.\pm1.t)=0.\quad t\in(0.2)$$

That

$$\boldsymbol{f_3=\Gamma\left(\frac{7}{8}\right)t\left(-\frac{35}{256}\sin(\pi x)\sin(2\pi y)+i\frac{105}{512}\sin(2\pi x)\sin(\pi y)\right)}$$

$$+t^{15/8}\left(-\frac{5}{8}\sin(2\pi x)\sin(\pi y)-i\frac{5}{12}\sin(\pi x)\sin(2\pi y)\right)$$
$$+t^{45/8}\left(\frac{1}{64}\sin^2(2\pi x)\sin^2(\pi y)+\frac{1}{144}\sin^2(\pi x)\sin^2(2\pi y)\right)$$
$$\cdot\left(\frac{1}{8}\sin(2\pi x)\sin(\pi y)+i\frac{1}{12}\sin(\pi x)\sin(2\pi y)\right) \quad (65)$$

It is proved that the exact solution of this equation is as follows:

$$\psi(x.y.t)=t^{15/8}\left(\frac{1}{8}\sin(2\pi x)\sin(\pi y)+i\frac{1}{12}\sin(\pi x)\sin(2\pi y)\right) \quad (66)$$

As we see result in the figure (3) and (4).

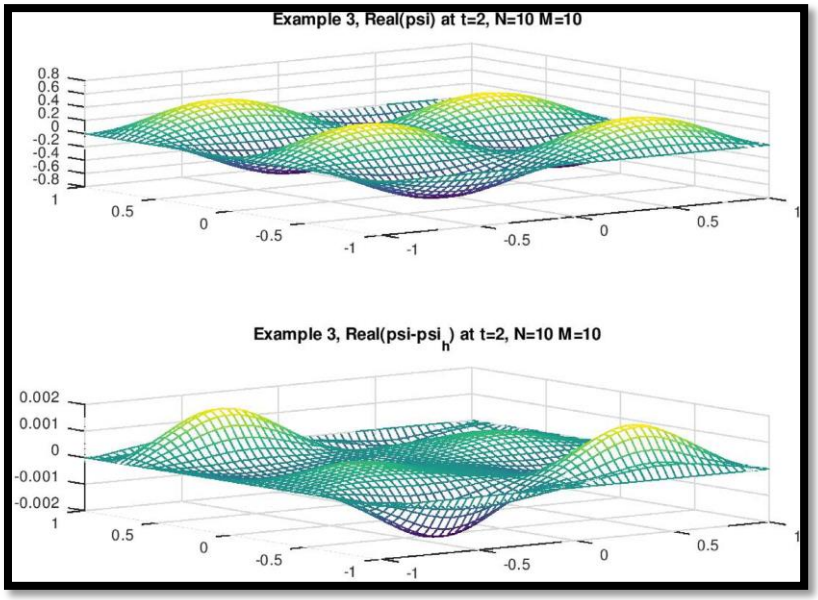

Figure (3): The approximate diagram of the numerical solution related to example 3 in a special case

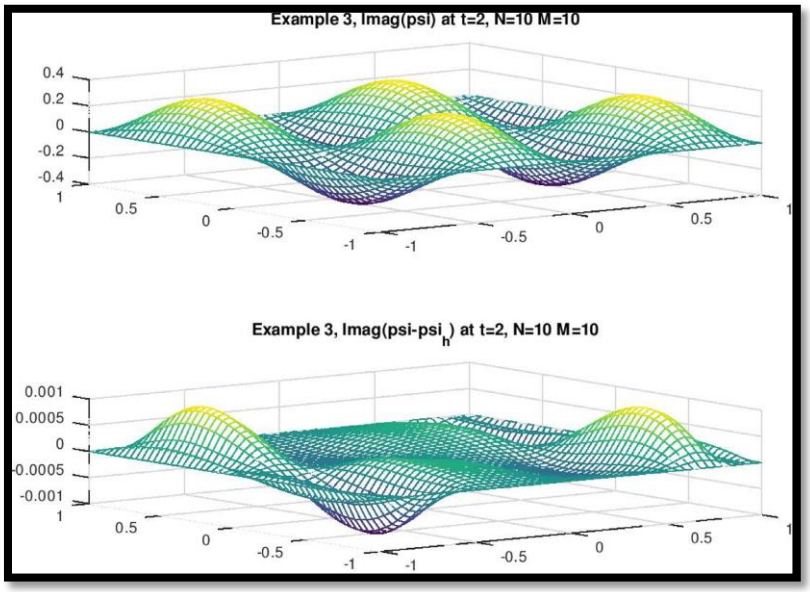

Figure (4): The approximate diagram of the numerical solution related to example 3 in a special case

4.2 Examples of numerical solution and results analysis

In this section, we use the proposed method for a non-linear fractional differential equation system of type one.

Example 4. First, we consider the following Volterra nonlinear fractional differential equation system.

$$\begin{cases} D_*^{\alpha_1} y_1(t) = g_1(t) + y_3(t) + \int_0^t \frac{s t y_1(s) + y_2(s)}{\sqrt{t-s}} ds. & y_1(0) = 0 \\ D_*^{\alpha_2} y_2(t) = g_2(t) + y_1(t) + \int_0^t \frac{y_2(s) + y_3(s)}{\sqrt[3]{t-s}} ds. & y_2(0) = 0 \\ D_*^{\alpha_3} y_3(t) = g_3(t) + y_3(t) + \int_0^t \frac{y_1(s) + s^2 t y_2(s)}{\sqrt[4]{t-s}} ds. & y_3(0) = 0 \end{cases} \quad (67)$$

Example 5. At the end of the apparatus, we consider the following non-linear fractional differential equations of Volterra.

$$\begin{cases} D_*^{\frac{3}{4}} y_1(t) = g_1(t) + y_2(t) + \int_0^t \frac{y_1(s) + [y_2(s)]^2}{\sqrt{t-s}} ds. & y_1(0) = 0 \\ D_*^{\frac{1}{2}} y_2(t) = g_2(t) + [y_1(t)]^3 + \int_0^t \frac{y_1(s) + y_2(s)}{(t-s)^{\frac{2}{3}}} ds. & y_2(0) = 0 \end{cases} \quad (68)$$

As we see result in the figure 5, 6,7,8,9,10 and 11

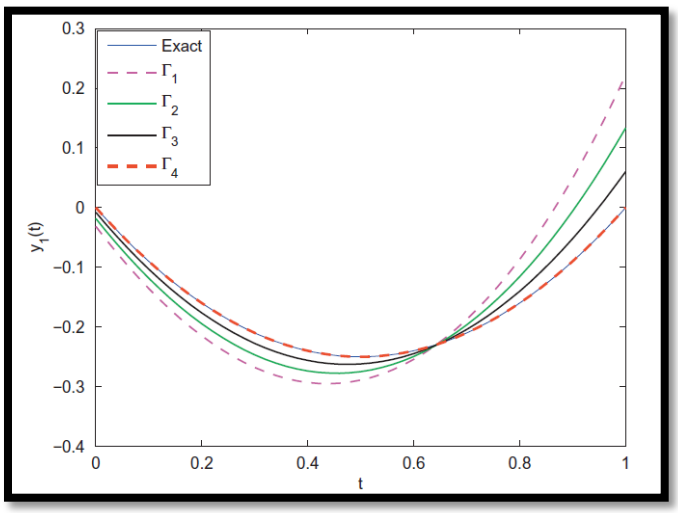

Figure (5): Numerical and approximate solution of the equation for different values of α

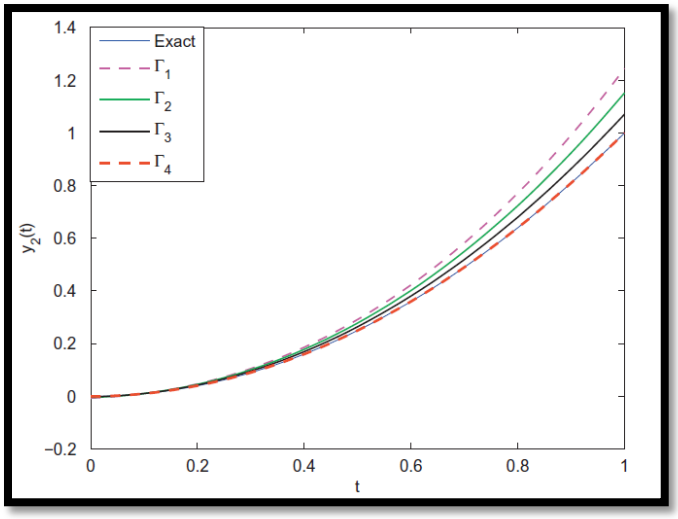

Figure (6): Numerical and approximate solution of the equation for different values of α

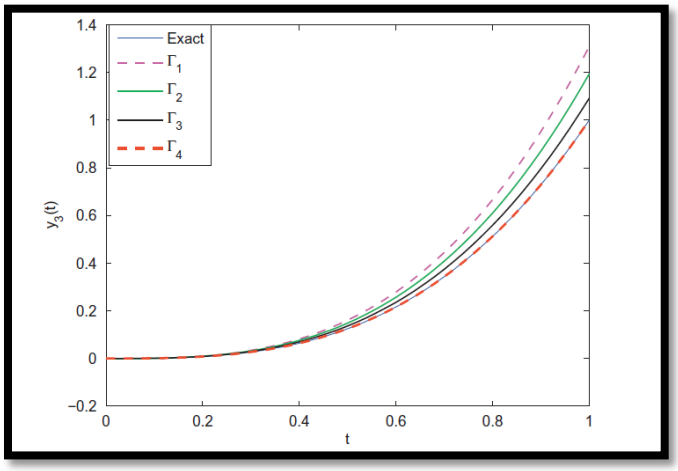

Figure (7): Numerical and approximate solution of the equation for different values of α

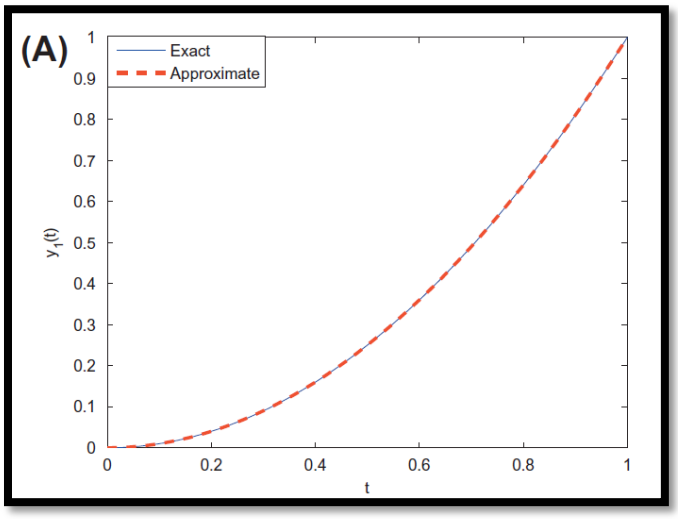

Figure (8): Numerical and approximate solution output

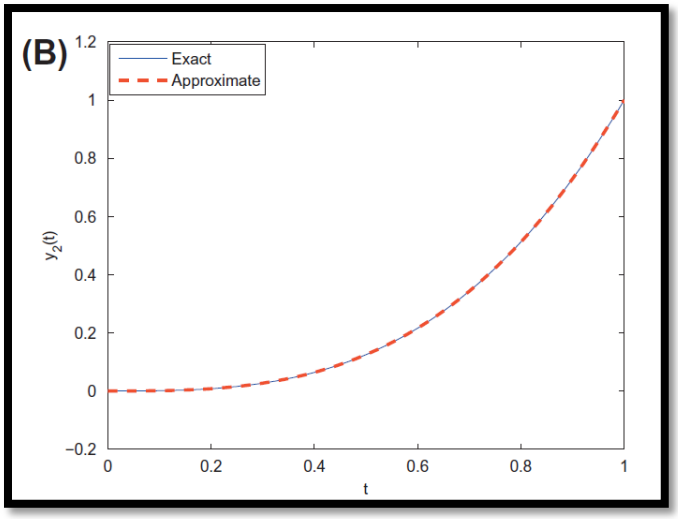

Figure (9): Comparison between numerical and exact answers

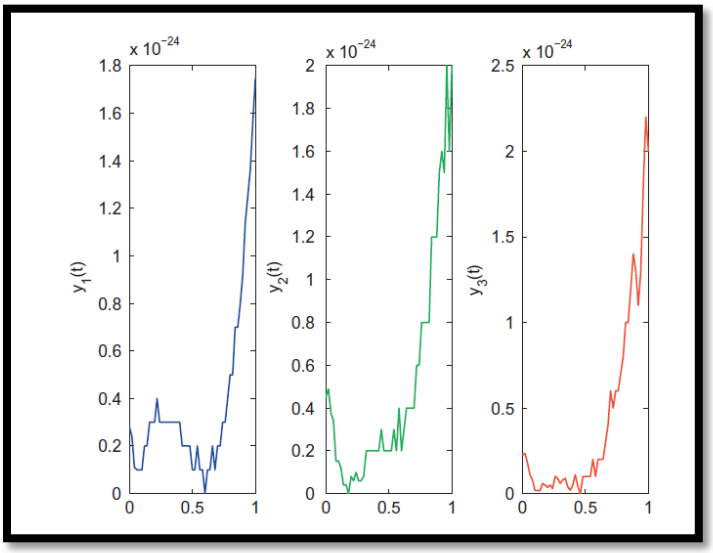

Figure (10): Output graphs

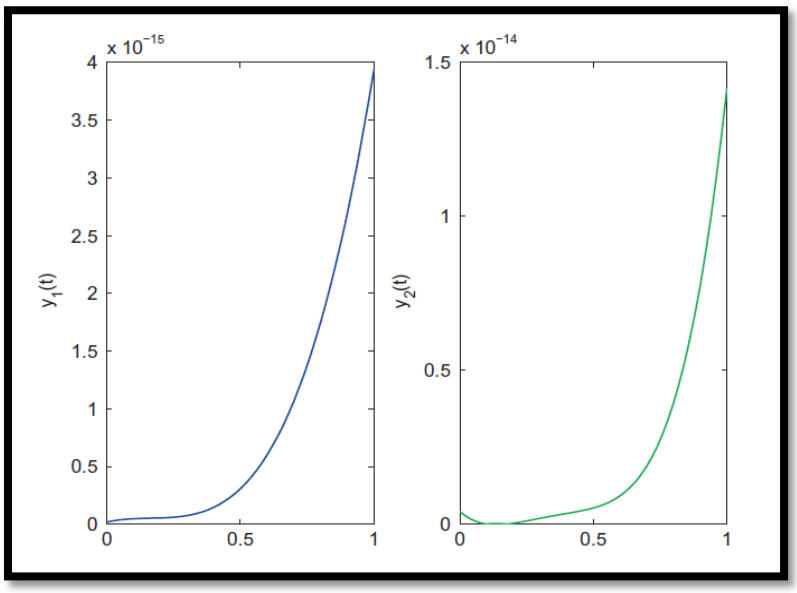

Figure (11): Absolute error graph between numerical and exact answers

5. CONCLUSION

Different approaches have been put out thus far to study fractional differential equations. We deal with derivatives and integrals of arbitrary order (and not always the proper order) in differential and integral calculus. Fractional derivatives have been used in a great deal of scientific and technical challenges in recent years. Numerous writers have reviewed applications of integral and differential calculus. For instance, the nonlinear oscillation model of an earthquake is studied using differential calculus and fractional integral. This technology is also used by dynamic traffic models. These concepts are applicable to many different areas, including as economics, signal processing, control theory, solid mechanics, statistical chain mechanics, and the oscillatory behavior of many elastic materials.

In this study, one-dimensional and two-dimensional fractional time Schrödinger equations are solved with suitable starting and boundary conditions using the JSC space-time approach. To do this, one must first translate the original problem into the matching set of integral differential equations that are weakly nonlinear., and then reducing the original equation to the corresponding equation by applying the integration scheme in conjunction with the high-precision Gaussian square law approximation of the existing integrals. the nonlinear algebraic equation system. The Newton-Raphson iterative technique is one powerful iterative solution that may be used to solve this system of algebraic equations. Based on current numerical techniques, the method that is being given offers two main benefits.. The proposed method demonstrates spectral accuracy in temporal and spatial variables and is well-balanced.. Furthermore, a thorough convergence study is offered to mathematically justify the suggested numerical concept. The numerical experiment results indicate that the suggested spectral approach calculates high precision solutions even with a small number of harmonic points. It is now recommended that this method be applied and expanded in order to address further time fractional differential equations. However, the aforementioned approach should be modified appropriately for this purpose.

Apart from the perspectives put forward in this thesis, we have the option to estimate fractional differential equation solutions using alternative forms of discretization of continuous functions. This may be achieved by utilizing the wavelet types and fundamental functions covered in functional and numerical analysis. One well-known fundamental function that we may utilize to accurately approximate continuous functions is the Chelyshkov polynomial.

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