



The Effect of Energy on (F-X-D) Types of Fungi, with Exponential Function

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Abstract

In my paper learning growth of types of fungi when mix two types Branching, Tip death. These types are Consumed all the energy, i.e. here the energy equals one. This biological phenomenon is represented as mathematical model as partial differential equations (PDEs). In study need, which is the fact for the evolution fungi, Solution of system depended on numerical solution and this solution gives approximation solution. Some steps on this solution as steady states, phase plane, travelling wave solution and using code to solve it when determent the initial condition after that we show the behaviour of growth of fungi.

Keywords: Branching, Tip dearh, Energy, Exponential function, hyphal death.

تحليل الطاقة على نوع (F-X-D) من الفطر باستخدام الدالة الاسية
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Paper Info.

Published: June 2023

الخلاصة

في هذا البحث نصف نمو الشاذ والنماذج الرياضية لنبات الفطر ، ان هذا النموذج يوضح السلوك لنمو التفرع الثنائي للفطر ، وموت الاطراف بسبب الاكتظاظ هو نموذج متكون من معادلة تفاضلية جزئية مع اضافة الطاقة . وكذلك نبين استهلاكها من قبل النبات ، بشكل عام أن نمو الفطريات يحتاج إلى حل للتمكن من انتاج افضل الانواع وذلك باقل وقت وجهد وكلفه من خلال التنبؤ بافضل انواع النباتات للزراعة وافضلها واقلها وقت وتكلفة بواسطة النتائج التي نحصل عليها . من خلال توصلنا الى حل رياضي باستخدام حل المعادلات التفاضلية الجزئية والتحليل العددي ، واستخدمنا بعض الأكواد في برنامج ماتلاب في التحليل العددي بسبب بعض الصعوبات التي نواجهها في الحل الرياضي المباشر. وبالتالي ستوضح نتائج هذا الحل نجاح او فشل نمو الفطريات المدروسة.

الكلمات المفتاحية: تفرع الجانبي ، الفروع الميتة ، الطاقة ، الدالة الاسية ، معدل النمو التفرع

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معلومات البحث

تأريخ النشر : حزيران 2023

Introduction

In 1982, Leah-Keshed denoted that Lateral Branching (F), Tip dearh was due to overcrowding (X), and Hyphal death (D) [1, 2].

There are many papers of the mathematical models which have been proposed by various

researchers in order to explain the Mathematical Model, for example:

-In (2011) Shuaa [2], Studied to develop a model for the growth of fungi which can be used to create a source term in a single root model to account for nutrient uptake by the fungi.

Therefore, there is a focus on the hyphal loss or death.

-In (2012) Brian Ingalls [3], offered an introduction to mathematical concepts and techniques needed for the construction and interpretation of models in molecular systems biology.

-In (2013) Walter [7], Studied independent sections that illustrate the most important

principles of mathematical modeling, a variety of applications, and classic models...

-In (2014) Mudhafar [8], Proposed the different modelling procedures, with a special emphasis on their ability to reproduce the biological system and to predict measured quantities which describe the overall processes. A comparison between the different methods is also made, highlighting their specific features.

Table (1) below illustrates these types.

Table (1): Clarify branching, biological, code of this type and version

Biological	Symbol	Version	Version Description
Lateral Branching	F	$\delta = \alpha_2 \eta$	α_2 denotes the number of branches produced per unit length of hypha per unit time.
Tip-death-dur-to overcrowding	X	$\delta = -\beta_3 p^2$	β_3 Is the rate at which density overcrowding limitss
Hypal death	D	$d = \gamma_1 p$	γ_1 Is the hyphal loss is rate high (constant for the death)

Reference: In 1982, Leah-Keshed denoted that [3].

Mathematical model

We investigated a new type fungal growth branching with thread death and consumption of whole plant food, which we can call energy E (ψ). This energy function lies between one and zero, with $0 \leq E(\psi) \leq 1$ indicating that the growth dies if

it does not consume energy. However, E (ψ) indicates that the growth is good if the fungi consumes all of the energy. [1, 2, 4].

We can describe the growth of Fungi by the equations below:

$$\frac{\partial \rho}{\partial t} = n\nu - d\rho \tag{1}$$

$$\frac{\partial n}{\partial t} = -\frac{\partial(n\nu)}{\partial x} + e^{[\delta(p,n)]} - E(\psi)$$

Where: $\delta(p, n) = \alpha_2 p - \beta_3 p^2$ that is dented above and $E(\psi) = 1$. Then this system (1) becomes: [2]

$$\frac{\partial \rho}{\partial t} = n\nu - \gamma\rho \tag{2}$$

$$\frac{\partial n}{\partial t} = -\frac{\partial(n\nu)}{\partial x} + e^{[\alpha p(1-p)]} - 1$$

Where: $\alpha = \frac{\alpha_2 \nu}{\gamma_1^2}$

Non-dimensionlision and Stabilit

By Keshet in (1982) [2], and Ali H. in (2011), Demonstrate how these parameters can be positioned as lower dimensions

$$\begin{aligned} \frac{\partial \rho}{\partial t} &= n\nu - d\rho \\ \frac{\partial n}{\partial t} &= -\frac{\partial(n\nu)}{\partial x} + e^{[\alpha p(1-p)]} - 1 \end{aligned} \tag{3}$$

Where: $\alpha = \frac{\alpha_2 \nu}{\gamma_1^2}$ is the parameter α , It represents the hyphenated branching rate per unit length per unit time. $\alpha p (1 - p)$ thus represents the number of branches produced per unit time per unit hyphen length. [2, 3]

Now, to find steady states when take from system (2):

$$\frac{\partial \rho}{\partial t} = n - \rho = 0 \rightarrow n = \rho \tag{4}$$

And on the other hand

$$\frac{\partial n}{\partial t} = e^{[\alpha p(1-p)]} - 1 = 0 \rightarrow e^{[\alpha p(1-p)]} = 1$$

$$\ln[e^{[\alpha p(1-p)]}] = \ln[1] \rightarrow \alpha p(1 - p) = 0 \tag{5}$$

$\alpha p = 0,$ then $\rightarrow (p, n) = (0,0)$ and $(1 - p) = 0$

$p = 1,$ then $(p, n) = (1,1)$

So that we get two steady states $(p,n) = (0,0)$ and $(p,n) = (1,1)$ therefor, we take Jacobain of these equations. [5, 8, 6]

$$J_{(p,n)} = \begin{bmatrix} -1 & 1 \\ \alpha(1 - 2p) & 0 \end{bmatrix}$$

We can classify the critical point according to the eigenvalues of this matrix. Jacobain at (0, 0):

$$J_{(0,0)} = \begin{bmatrix} -1 & 1 \\ \alpha & 0 \end{bmatrix}$$

Thus, $|A - \lambda I| = 0$ we get two values of (λ):

$$\lambda_1 = \frac{-1}{2} + \frac{\sqrt{4\alpha+1}}{2}, \quad \lambda_2 = \frac{-1}{2} - \frac{\sqrt{4\alpha+1}}{2}$$

Then we take the Jacobain at (1, 1):

$$J_{(1,1)} = \begin{bmatrix} -1 & 1 \\ -\alpha & 0 \end{bmatrix}$$

Thus $|A - \lambda I| = 0$, then we get two values of (λ):

$$\lambda_1 = \frac{-1}{2} + \frac{\sqrt{-4\alpha + 1}}{2}, \quad \lambda_2 = \frac{-1}{2} - \frac{\sqrt{-4\alpha + 1}}{2}$$

We note the probabilities of the α . If α is negative, we get the point (0, 0) stable spiral and

(1,1) saddle point see Figure (1). Using (MATLAB pplane7) [4, 7, 6]

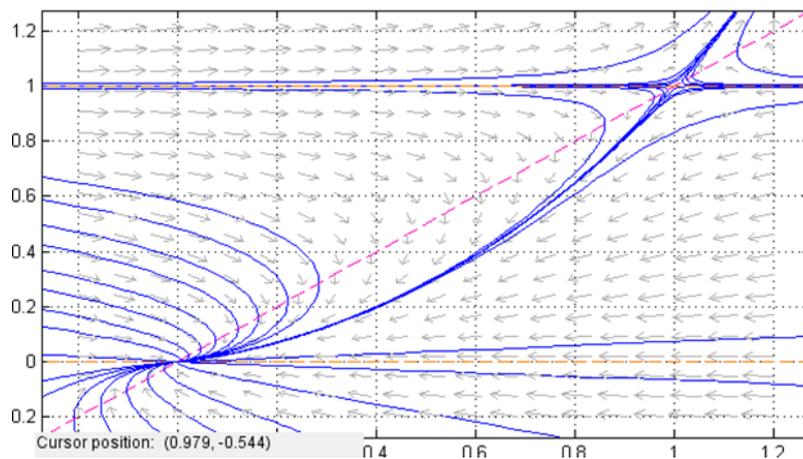


Figure (1): Note the plane (p, n) that the path connects at the point (1, 1) is the saddle point and at the point (0, 0) the stable node

Travelling wave solution

Now we will talk about the travelling wave solution, so that we consider that: $\rho(x, t) = \rho(z)$ and $n(x, t) = N(z)$ where $z = x - ct$, $P(z)$ profile

density and propagation rate c for the edge of the colony. $P(z)$ and $N(z)$ are a nonnegative function to Z . The function, $p(x,t)$, $n(x,t)$ are moving waves, moving at a constant speed c in the positive

x direction, where $c > 0$, $E(\psi) = 1$, and $\alpha = 1$ to search for the traveling wave solution of the

equations in x and t of the system (3).

$$\frac{d\rho}{dt} = -c \frac{d\rho}{dx}, \quad \frac{dn}{dt} = -\frac{dn}{dx}, \quad \text{And, } \frac{dn}{dt} = \frac{dN}{dx}$$

See [3] therefor, the above equation becomes:

$$\begin{aligned} \frac{dP}{dz} &= \frac{-1}{c}[N - P] \\ \frac{dN}{dz} &= \frac{1}{1-c} e^{[\alpha p(1-p)]}, \quad c \neq 1, \quad -\infty < z < \infty \end{aligned} \tag{6}$$

To deter the stability of the above system, then we get $(\eta, \rho) = (0,0)$ saddle point , and $(1,1)$ stable spiral constant for negative c and $\alpha = 1$. This

helps us determine the initial conditions to p and n which is the above system (3) see figure(2) below.

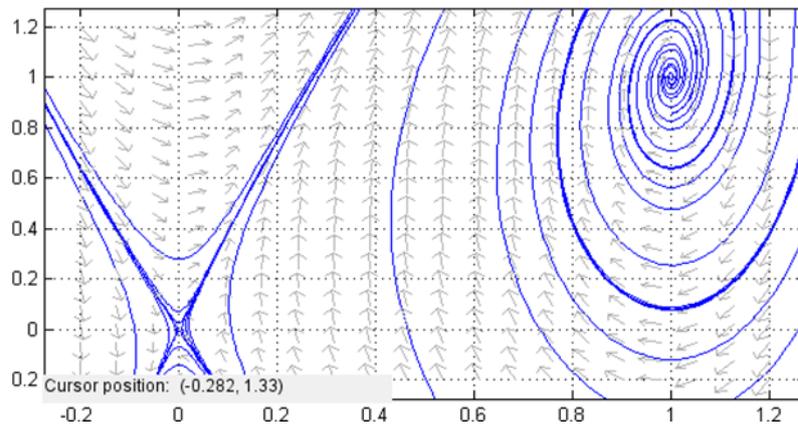


Figure (2): Note the plane (p, n) that the path connects at the point (0, 0) is the saddle point and at the point (1, 1) the stable spiral.

Numerical Solution

Because the system (3) cannot be solved exactly, so we resort to numerical solutions, and here we are using pdepe code in MATLAB .

If we notice that the initial condition starts from 1 to zero, we also notice that Fig. (3) behaves ρ and n it is clear that the traveling waves travel uniformly through time.

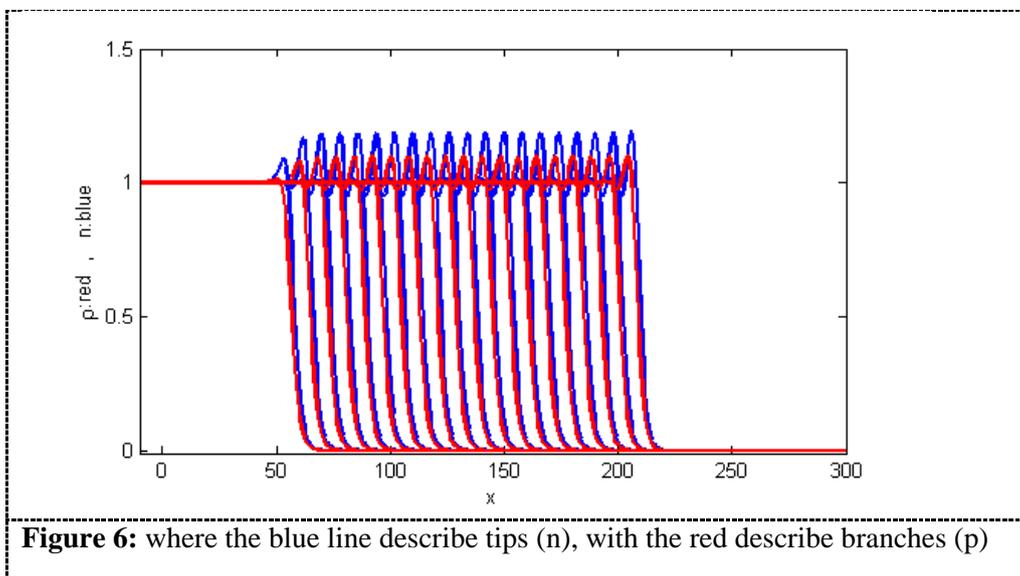
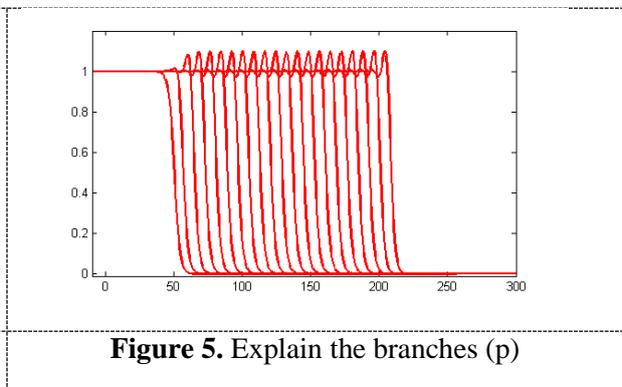
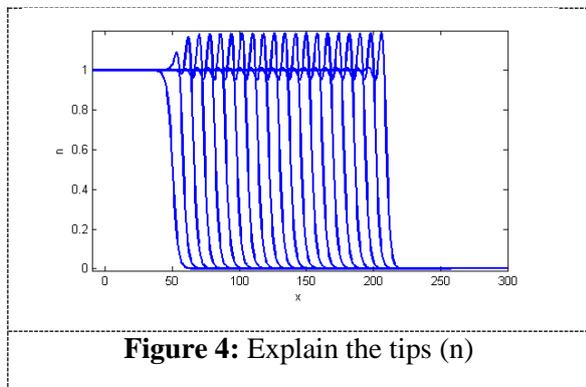
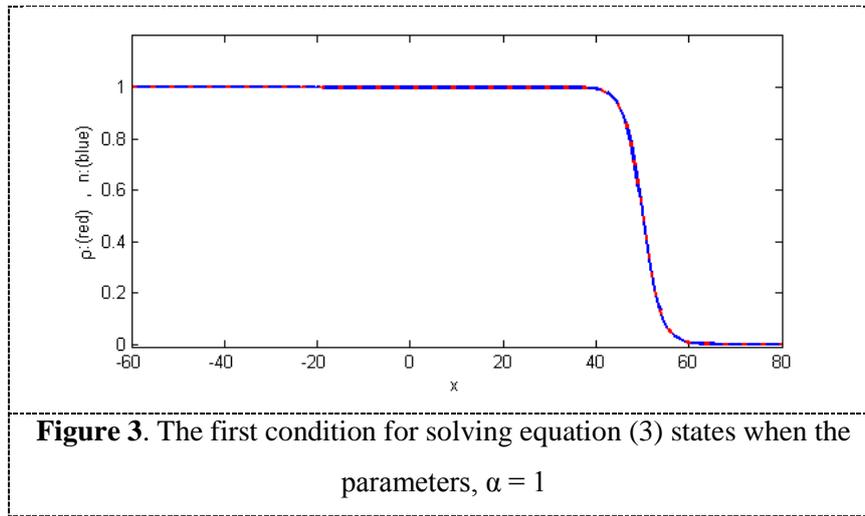
Illustrate solution to the system (3) with the parameters $\alpha = 0.5$

And $c = 1.594$ for time $t = 1, 10, \dots, 300$.

In (Figure 4) below blue line describes tips (n),

In (Figure 5) red line describes branches (p), In (Figure 6) where the blue line describes tips (n), with the red which describes branches (p), and illustrates the solution of ρ and n numerically with take values of $\alpha = 0.5$, that is so clear the traveling wave s solution begging from left to right and still the same tidal wave.

From all of this we conclude, we get the relationship between the values of the moving waves c and α by taking $v = d = 1$, by using Matlab [2,7,8].



From all of this we conclude, we get the relationship between the values of the moving waves c and α by taking $v = d = 1$, we can show

that Table (2). Where α increases from the traveling wave solution C , (see Fig. 7).

Table 2: Correlation between the speed of c waves and the values of α with taking $v = d = 1$

Reference : Results of program code in Matlab

α	0.5	1	2	3	4	5	6	7	8	9
c	1.59	2.04	2.66	3.03	3.55	4.04	4.53	5.12	5.74	6.09

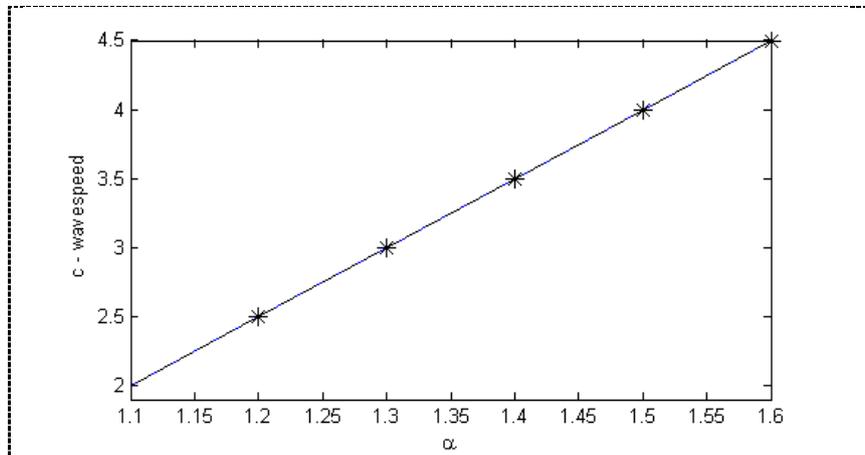


Figure 7: Correlation between the speed of c waves and the values of α with taking $v=d=1$

Now, we get the relationship between the values of the transmitted wave c and by taking $\alpha = d = 1$, as shown in Table (3) where v remains constant,

The increase of resolving C travelling waves is increasing. See Figure (8).

Table 3: Correlation between the speed of c waves and the values of v with taking $v = d = 1$

v	0.5	1	2	3	4	5	6	7	8	9
c	1.60	2.04	2.71	3.37	3.88	4.31	4.71	5.11	5.69	6.11

Reference : Results of program code in Matlab

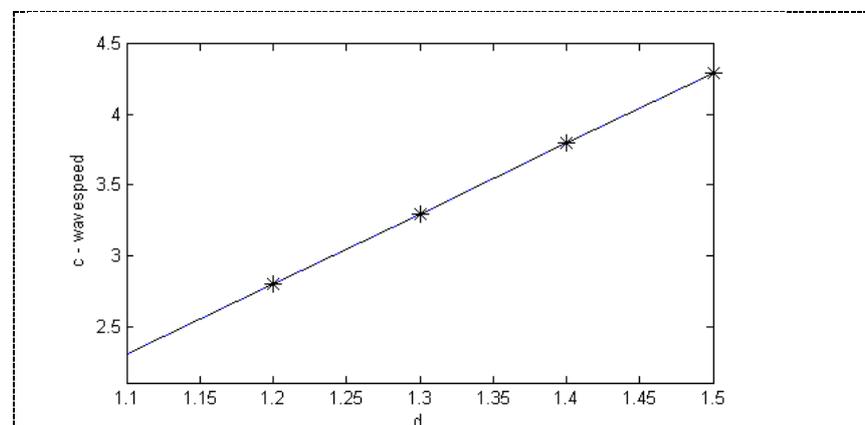


Figure 8: Correlation between the speed of c waves and the values of v with taking $\alpha=d=1$

Then, we get the relationship between the values of the transmitted wave c and by taking $\alpha = v = 1$, as shown in Table (4) below where d remains

constant, The increase of resolving C traveling waves is increasing. See Figure (9).

Table 4. Correlation between the speed of c waves and the values of d with taking $v = \alpha = 1$

d	0.5	1	2	3	4	5	6	7	8	9
c	4.87	4.41	2.71	3.85	3.61	1.62	1.80	1.01	0.92	0.43

Reference : Results of program code in Matlab

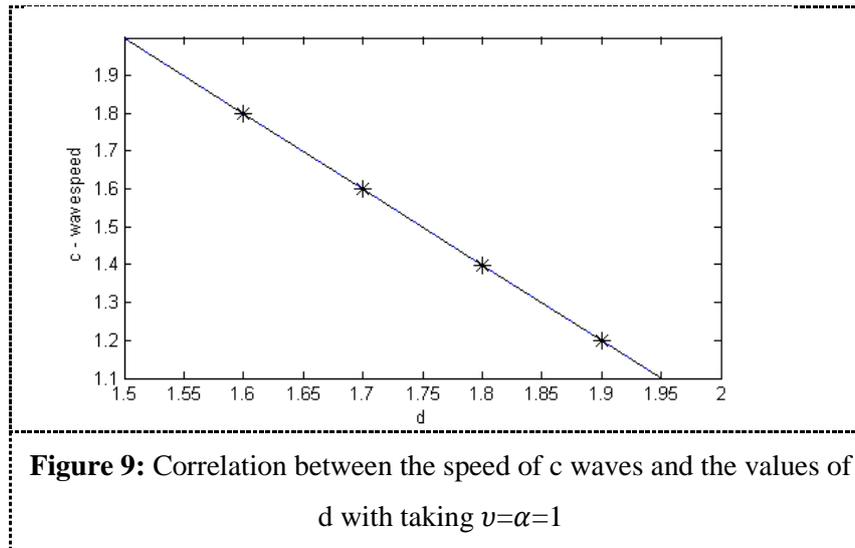


Figure 9: Correlation between the speed of c waves and the values of d with taking $v=\alpha=1$

Conclusion

Through our study, we obtained a relationship between c and α , see **Figure 3**, if you notice that the wave velocity c increases when α is an increasing function of α . It is also clear that the relationship between c and d , then c decreases as d increases. See **Figure 3**, and we plot the relationship between c and, this is clear that the speed of wave c increases when v increases.

Since ($\alpha = \frac{\alpha_2 v}{\gamma_1}$), therefore the growth rate is always increasing with α_2 while keeping and γ_1^2 is fixed, also the growth rate is always decreasing with γ_1^2 increases while keeping α_2 and v are fixed. We note that changing the rate of anastomosis β_2 , cannot alter the colony growth rate, since the growth parameter α has no

dependence on β_2 . However, increasing β_2 word decrease the density levels accumulated in the interior see table (1) [3, 2].

α_1 = Is the number of tips produced per tip per unit time.

γ_1 = Is the loss rate of hyphal (constant for hyphal death).

β_2 = Is the rate of tip reconnections per unit length hypha per unit time.

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